

Supplementary Information

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In summary, the process to determine if a musical network has the scale-free property according to Clauset's statistical method comprises three steps: (i) power law α exponent estimation, (ii) Kolmogorov-Smirnov p -value test, and (iii) likelihood ratio tests. A musical network has the scale-free property if $2 < \alpha < 3$; and if p -value > 0.1 ; and if the likelihood ratio tests confirm power law as the best distribution compared to alternative distributions (exponential, stretched exponential and log-normal).

The process to determine if a musical network has the small-world property was quite simple, because almost all musical networks presented average clustering coefficient (ACC) greater than the equivalent random and small-world networks. So, we have calculated the average of all mean shortest path length (MSPL) among random and small-world networks. We have also calculated the standard deviation and the confidence interval of 95%. If (i) the MSPL result from the musical network is within the confidence interval; and, if (ii) its average clustering coefficient is close to or greater than the ACC result from the small-world networks, we consider this musical network compatible with the small-world property.

This document is organised as follows. First, the numerical results for the forty musical networks are presented in tables S1, S2, and S3. Afterwards, we give details of our analyses of each piece of music.

	Network Size		Clauset's First and Second Steps	
Musical Network	nº nodes	nº edges	power law exponent (α)	KS p-value
Bach BWV 801	180	239	2.9336	0.906
Bach BWV 1001	413	1107	2.16431	0.112
Bach BWV 1003	549	1394	2.12844	0.214
Bach BWV 1004	681	1715	1.98763	0.004
Bach BWV 1005	620	1469	1.97255	0.184
Bach BWV 1041	1560	2598	2.2254	0.482
Beethoven Opus 10	1376	2151	2.54803	0.304
Beethoven Opus 13	1645	2395	2.64127	0.132
Beethoven Opus 22	1880	3175	2.31242	0.083
Beethoven Opus 27	1159	1835	2.30737	0.100
Beethoven Opus 53	2259	3707	2.25062	0.028
Beethoven Opus 57	1854	3206	2.1937	0.590
Beethoven Opus 81	1432	2317	2.09344	0.304
Beethoven Opus 90	1118	1720	2.37218	0.206
Beethoven Opus 106	4818	7907	2.18382	0.048
Brahms Opus 1	2903	4166	2.45447	0.504
Chopin Opus 35	1487	2319	2.60524	0.134
Clementi N°1	297	466	4.55607	0.794
Clementi N°2	308	494	2.38659	0.162
Clementi N°3	402	724	2.72179	0.000
Clementi N°4	425	787	2.17386	0.090
Clementi N°5	478	900	2.27392	0.270
Clementi N°6	477	777	2.35256	0.058
Haydn Hoboken XVI:9	290	464	2.74506	0.011
Haydn Hoboken XVI:33	721	1413	2.43118	0.285
Haydn Hoboken XVI:35	835	1560	2.38432	0.084
Haydn Hoboken XVI:40	642	1094	2.56768	0.132
Haydn Hoboken XVI:43	732	1440	2.26035	0.114
Mozart KV311	1234	2179	2.33052	0.222
Mozart KV330	947	1719	2.49826	0.223
Mozart KV331	1144	1977	2.64005	0.928
Mozart KV332	1340	2299	2.40332	0.424
Mozart KV333	1532	2815	2.08961	0.164
Mozart KV545	634	1107	2.6527	0.181
Mozart KV570	1052	1721	2.37106	0.392
Schubert D960	2768	4706	2.10174	0.008
Schubert D760	2493	3855	2.44852	0.070
Schubert D850	2948	4613	2.30162	0.332
Schubert D784	1349	1905	3.01568	0.342
Shostakovich Opus 57	1293	1842	3.06283	0.320

Table S1: Number of nodes, number of edges, α exponent, and p -value of musical networks.

Likelihood Ratio Test – Clauset's Third Step								
Musical Network	Power law (KS) <i>p</i> -value	Exponential		Log-normal		Stretched exponential		Scale-free?
		LR	<i>p</i> -value	LR	<i>p</i> -value	LR	<i>p</i> -value	
Bach BWV 801	0.906	3.40	0.00	2.73	0.00	2.13	0.03	YES
Bach BWV 1001	0.112	3.02	0.00	0.98	0.32	0.07	0.94	No
Bach BWV 1003	0.214	3.96	0.00	1.56	0.11	0.54	0.58	No
Bach BWV 1004	0.004	3.16	0.00	-0.03	0.97	-1.15	0.24	No
Bach BWV 1005	0.184	5.27	0.00	2.18	0.02	0.85	0.39	No
Bach BWV 1041	0.482	1.92	0.05	-0.34	0.73	-0.69	0.48	No
Beethoven Opus 10	0.304	6.35	0.00	4.71	0.00	3.74	0.00	YES
Beethoven Opus 13	0.132	5.76	0.00	4.44	0.00	3.55	0.00	YES
Beethoven Opus 22	0.083	10.37	0.00	7.76	0.00	5.56	0.00	No
Beethoven Opus 27	0.100	7.67	0.00	5.44	0.00	3.96	0.00	YES
Beethoven Opus 53	0.028	2.49	0.01	1.66	0.09	1.44	0.14	No
Beethoven Opus 57	0.590	9.88	0.00	6.36	0.00	4.09	0.00	YES
Beethoven Opus 81	0.304	10.63	0.00	7.66	0.00	5.82	0.00	YES
Beethoven Opus 90	0.206	4.17	0.00	3.31	0.00	2.47	0.01	YES
Beethoven Opus 106	0.048	5.78	0.00	0.41	0.67	-1.05	0.29	No
Brahms Opus 1	0.504	4.87	0.00	3.75	0.00	2.82	0.00	YES
Chopin Opus 35	0.134	6.03	0.00	4.08	0.00	2.62	0.00	YES
Clementi No.1	0.794	1.38	0.16	1.00	0.31	0.78	0.43	No
Clementi No.2	0.162	3.52	0.00	2.34	0.01	1.48	0.13	YES
Clementi No.3	0.000	4.91	0.00	3.84	0.00	2.55	0.01	No
Clementi No.4	0.090	-0.28	0.77	-0.42	0.67	-0.44	0.65	No
Clementi No.5	0.270	5.57	0.00	3.67	0.00	2.64	0.00	YES
Clementi No.6	0.058	5.84	0.00	4.30	0.00	3.21	0.00	No
Haydn Hoboken XVI:9	0.011	0.64	0.51	0.15	0.87	-0.29	0.76	No
Haydn Hoboken XVI:33	0.285	5.54	0.00	3.78	0.00	2.87	0.00	YES
Haydn Hoboken XVI:35	0.084	8.71	0.00	6.62	0.00	5.14	0.00	No
Haydn Hoboken XVI:40	0.132	0.67	0.49	0.39	0.68	0.36	0.71	No
Haydn Hoboken XVI:43	0.114	5.99	0.00	3.92	0.00	2.68	0.00	YES
Mozart KV311	0.222	5.28	0.00	2.83	0.00	2.00	0.04	YES
Mozart KV330	0.223	6.15	0.00	4.82	0.00	4.05	0.00	YES
Mozart KV331	0.928	4.18	0.00	2.88	0.00	2.31	0.02	YES
Mozart KV332	0.424	4.82	0.00	2.67	0.00	1.61	0.10	YES
Mozart KV333	0.164	2.44	0.01	-0.39	0.69	-0.91	0.36	No
Mozart KV545	0.181	4.49	0.00	3.28	0.00	2.71	0.00	YES
Mozart KV570	0.392	6.25	0.00	4.85	0.00	3.61	0.00	YES
Schubert D960	0.008	-0.35	0.72	-1.05	0.28	-1.11	0.26	No
Schubert D760	0.070	1.26	0.20	0.77	0.44	0.62	0.53	No
Schubert D850	0.332	6.32	0.00	2.97	0.00	1.74	0.08	YES
Schubert D784	0.342	6.32	0.00	5.42	0.00	4.27	0.00	YES
Shostakovich Opus 57	0.320	2.00	0.04	0.99	0.31	0.58	0.55	No

Table S2: Likelihood Ratio (LR) test for forty musical networks: power law vs. exponential, power law vs. log-normal, power law vs. stretched exponential.

Musical Networks			Random Networks		Small-world Networks		
	MSPL	ACC	MSPL	ACC	MSPL	ACC	Small-world?
Bach BWV 801	5.80	0.01	5.28	0.00	4.09	0.07	No
Bach BWV 1001	3.40	0.19	3.78	0.01	4.79	0.06	YES
Bach BWV 1003	3.37	0.27	4.05	0.00	5.06	0.07	YES
Bach BWV 1004	5.05	0.26	4.22	0.00	5.23	0.07	YES
Bach BWV 1005	3.60	0.21	4.31	0.00	5.20	0.07	YES
Bach BWV 1041	5.01	0.15	6.24	0.00	6.01	0.08	YES
Beethoven Opus 10	6.43	0.07	6.44	0.00	5.86	0.06	YES
Beethoven Opus 13	8.69	0.07	6.99	0.00	6.05	0.07	No
Beethoven Opus 22	6.43	0.15	6.53	0.00	6.12	0.06	YES
Beethoven Opus 27	8.02	0.12	6.29	0.00	5.68	0.07	No
Beethoven Opus 53	7.76	0.10	6.62	0.00	6.30	0.07	No
Beethoven Opus 57	5.88	0.18	6.22	0.00	6.08	0.06	YES
Beethoven Opus 81	6.02	0.15	6.24	0.00	5.91	0.07	YES
Beethoven Opus 90	7.47	0.13	6.32	0.00	5.68	0.06	No
Beethoven Opus 106	7.49	0.11	7.22	0.00	6.94	0.06	YES
Brahms Opus 1	9.33	0.07	7.55	0.00	6.53	0.07	No
Chopin Opus 35	12.64	0.09	6.50	0.00	5.95	0.08	No
Clementi No.1	6.37	0.14	5.14	0.01	4.51	0.07	No
Clementi No.2	4.98	0.09	5.02	0.00	4.54	0.07	YES
Clementi No.3	5.35	0.18	4.83	0.00	4.79	0.08	No
Clementi No.4	5.61	0.23	4.67	0.00	4.81	0.06	No
Clementi No.5	5.42	0.28	4.77	0.00	4.90	0.08	No
Clementi No.6	4.92	0.15	5.30	0.00	4.99	0.08	YES
Haydn Hoboken XVI:9	5.15	0.12	4.95	0.00	4.46	0.06	YES
Haydn Hoboken XVI:33	4.40	0.19	4.95	0.00	5.27	0.05	YES
Haydn Hoboken XVI:35	5.25	0.22	5.26	0.00	5.45	0.08	YES
Haydn Hoboken XVI:40	5.68	0.11	5.43	0.00	5.23	0.07	YES
Haydn Hoboken XVI:43	4.74	0.16	4.95	0.00	5.28	0.06	YES
Mozart KV311	4.60	0.12	5.80	0.00	5.82	0.07	YES
Mozart KV330	4.89	0.09	5.42	0.00	5.47	0.06	YES
Mozart KV331	5.24	0.11	5.78	0.00	5.74	0.08	YES
Mozart KV332	5.51	0.11	5.96	0.00	5.84	0.07	YES
Mozart KV333	5.02	0.18	5.77	0.00	5.96	0.06	YES
Mozart KV545	5.08	0.06	5.31	0.00	5.21	0.07	YES
Mozart KV570	5.70	0.06	5.96	0.00	5.60	0.06	YES
Schubert D960	6.28	0.10	6.54	0.00	6.48	0.06	YES
Schubert D760	9.07	0.10	7.08	0.00	6.39	0.07	No
Schubert D850	9.93	0.10	7.09	0.00	6.54	0.07	No
Schubert D784	13.67	0.06	7.10	0.00	5.91	0.08	No
Shostakovich Opus 57	9.68	0.05	6.93	0.00	5.86	0.08	No

Table S3: Mean shortest path length (MSPL) and average cluster coefficient (ACC) for musical networks, random networks, and small-world networks.

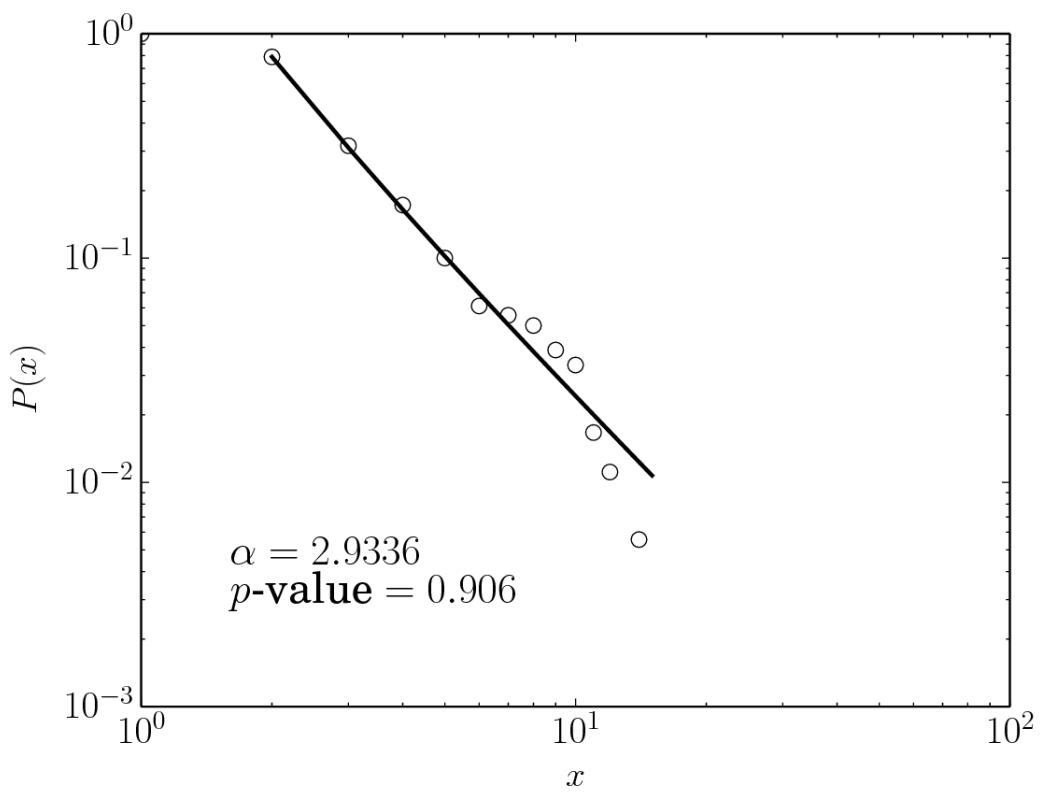


Figure 1: Bach - Sinfonia No. 15 in B minor, BWV 801 (1723)

→ Scale-free compatibility: YES (exponent α – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: NO (small ACC compared to small-world networks)

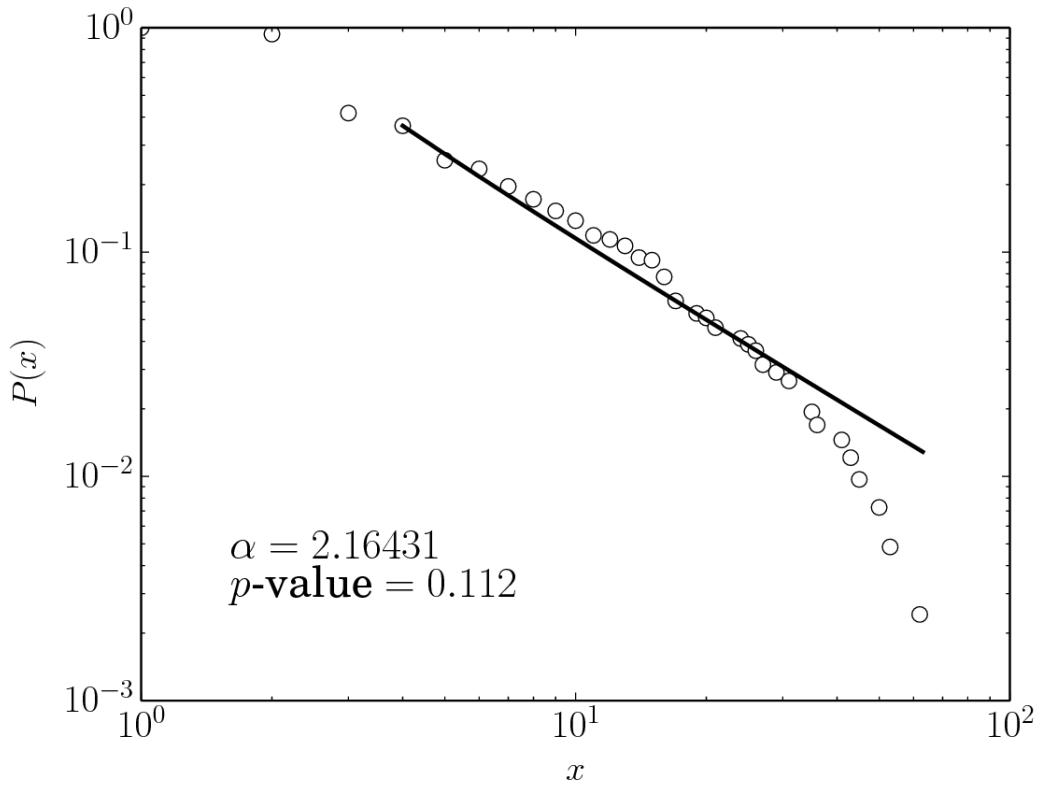


Figure 2: Bach - Violin Sonata No.1 in G minor, BWV 1001 (1720)

→ Scale-free compatibility: NO (p -value trend to log-normal or stretched exponential)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

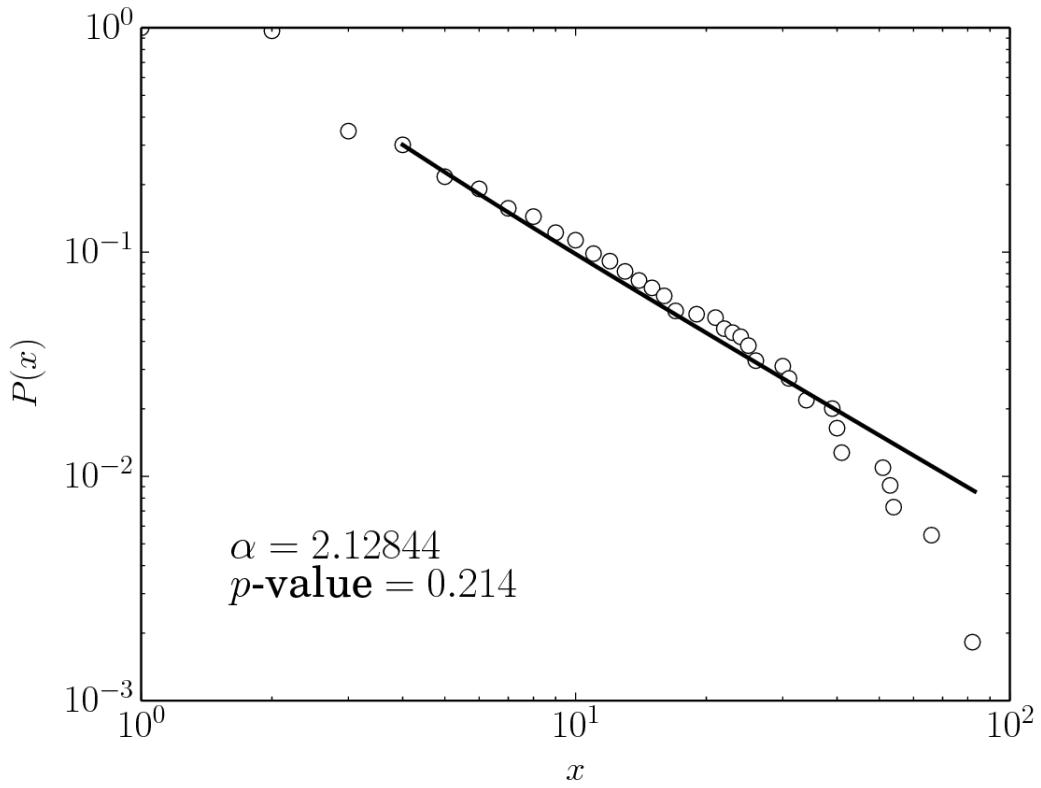


Figure 3: Bach - Violin Sonata No.2 in A minor, BWV 1003 (1720)

- Scale-free compatibility: NO (p -value trend to stretched exponential)
- Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

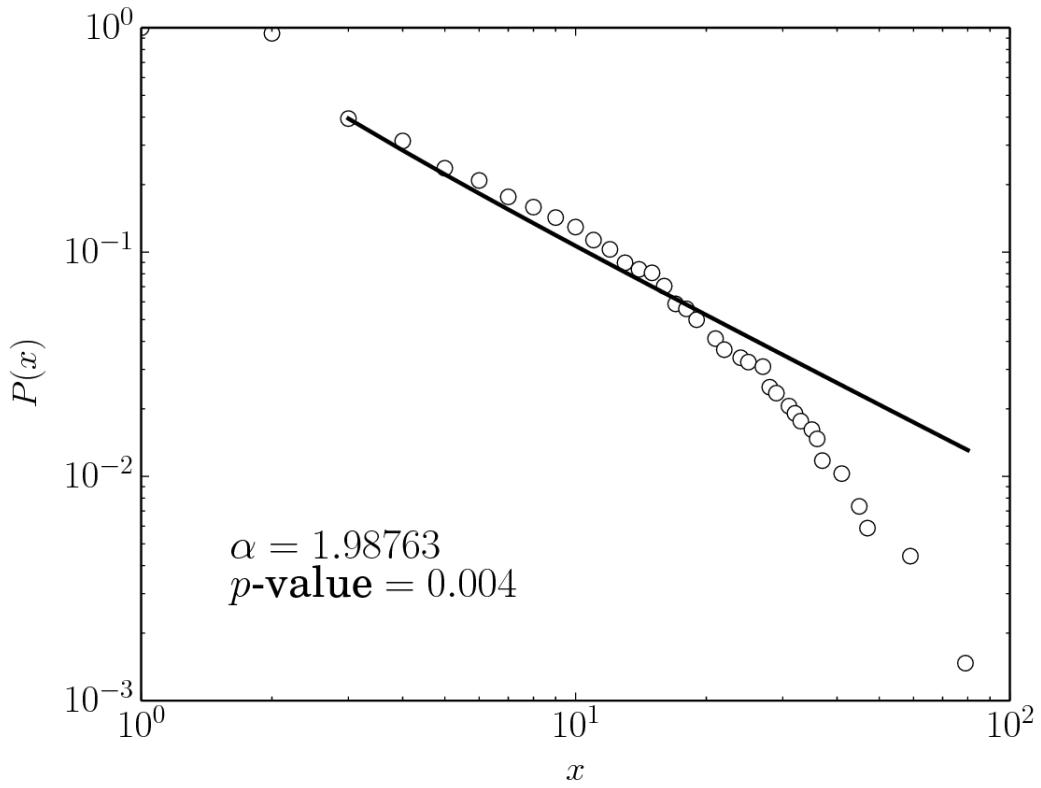


Figure 4: Bach - Violin partita No. 2 in D minor, BWV 1004 (1720)

→ Scale-free compatibility: NO (p -value trend to stretched exponential)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

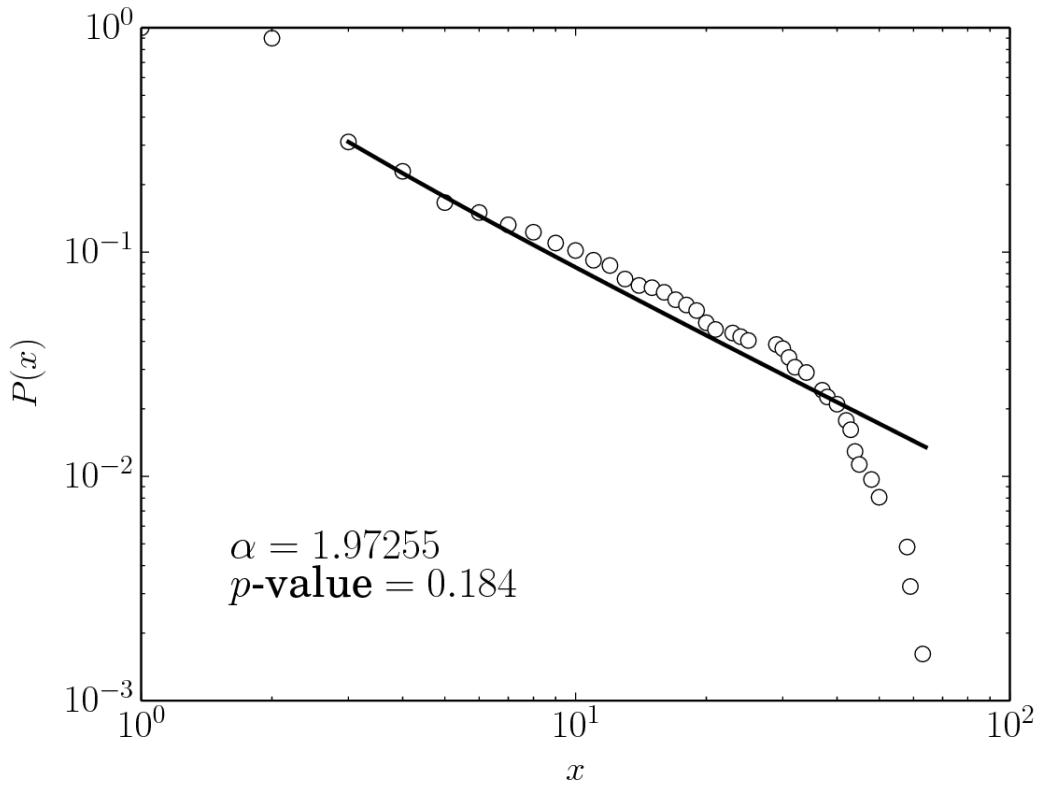


Figure 5: Bach - Violin Sonata No.3 in C major, BWV 1005 (1720)

- Scale-free compatibility: NO (p -value trend to stretched exponential)
- Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

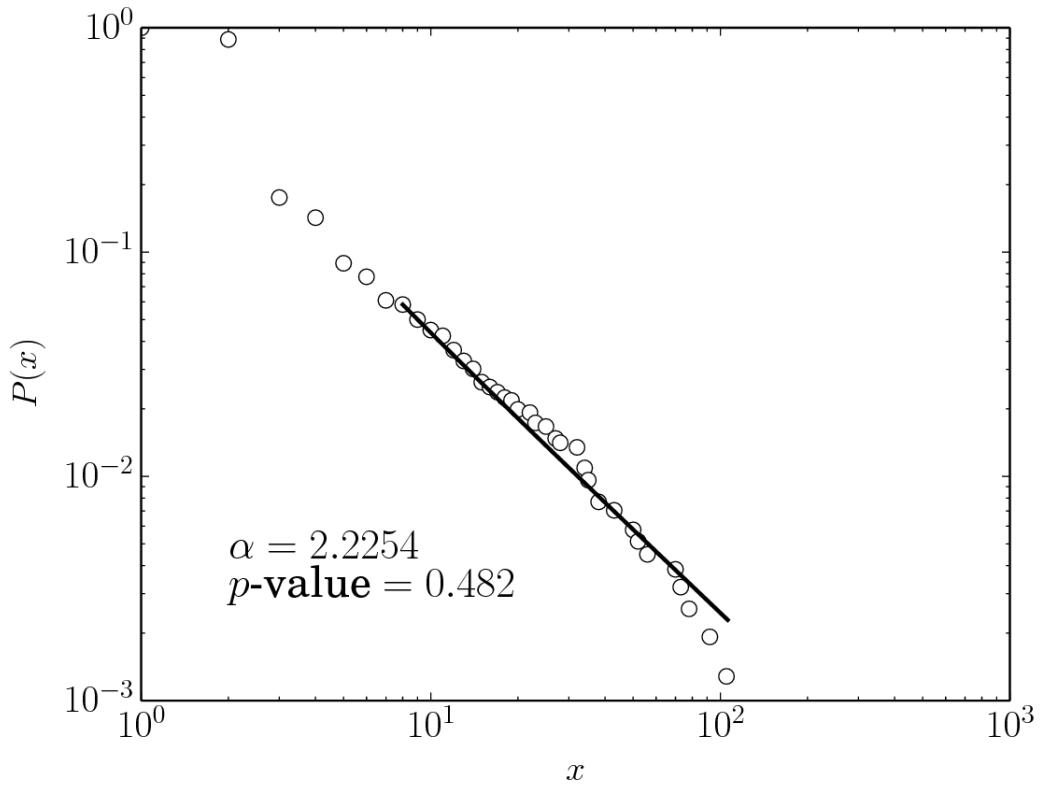


Figure 6: Bach - Violin Concerto in A minor, BWV 1041 (1917)

→ Scale-free compatibility: NO (p -value trend to log-normal)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

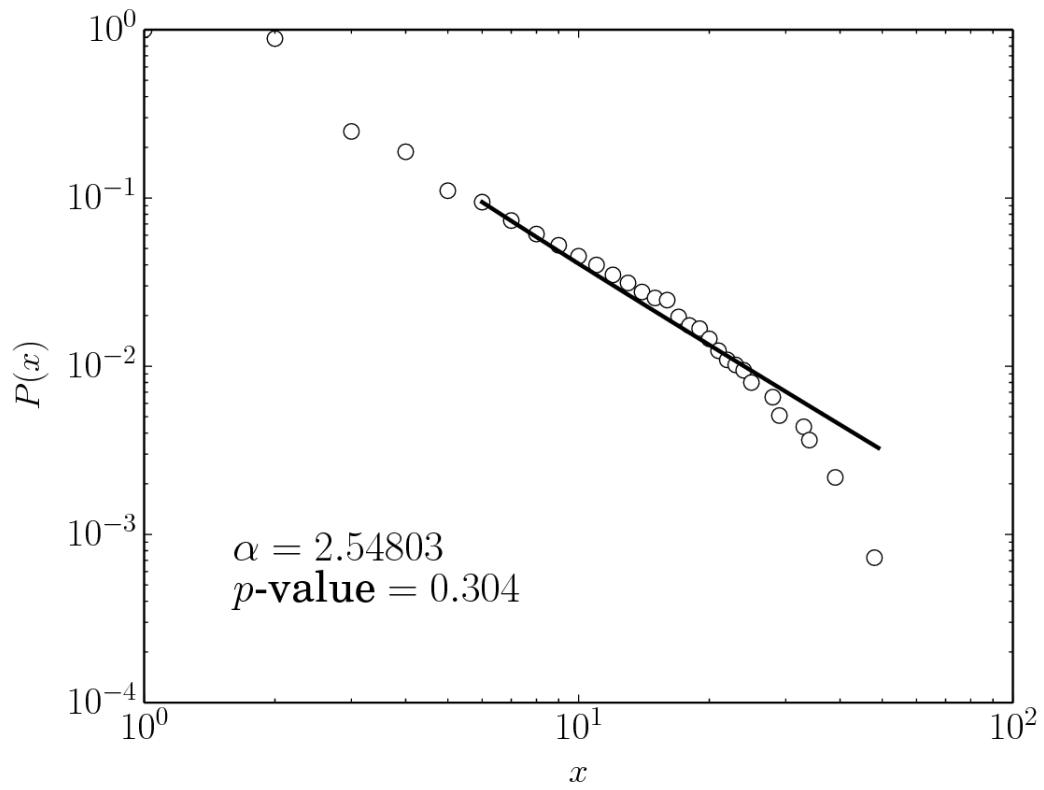


Figure 7: Beethoven - Sonata No. 5 in C minor, Opus 10/1 (1798)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

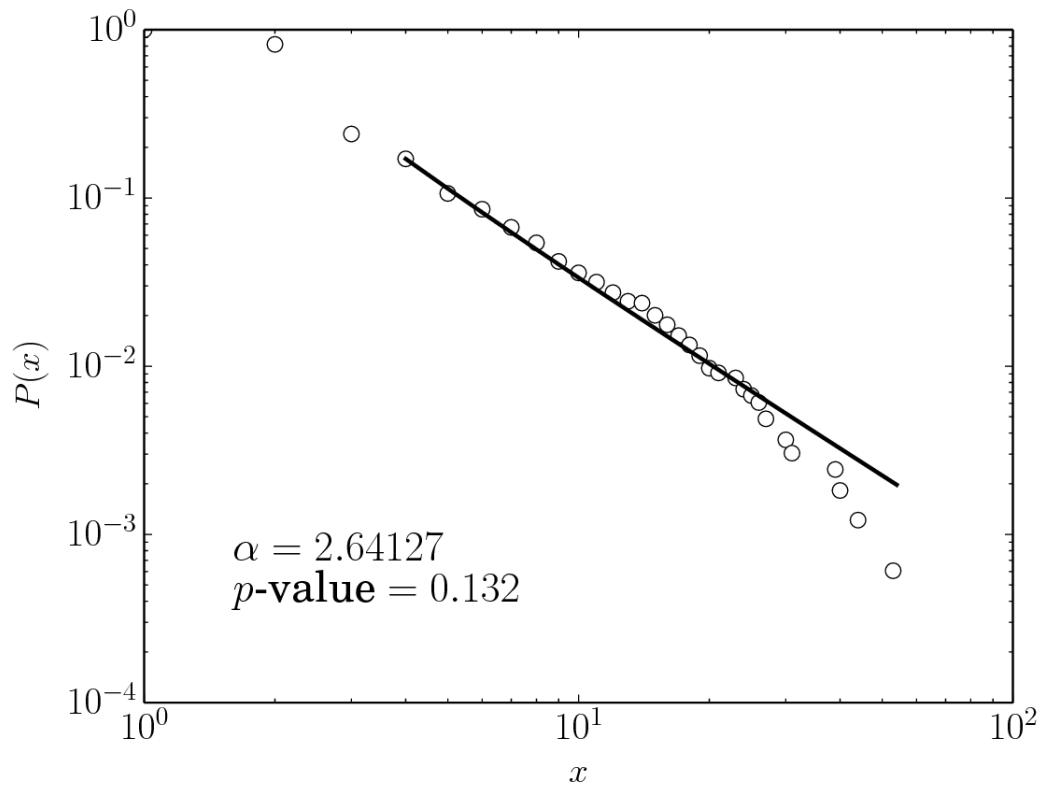


Figure 8: Beethoven - Sonata No. 8 in C minor (Pathetique), Opus 13 (1799)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: NO (MSPL outside the confidence interval)

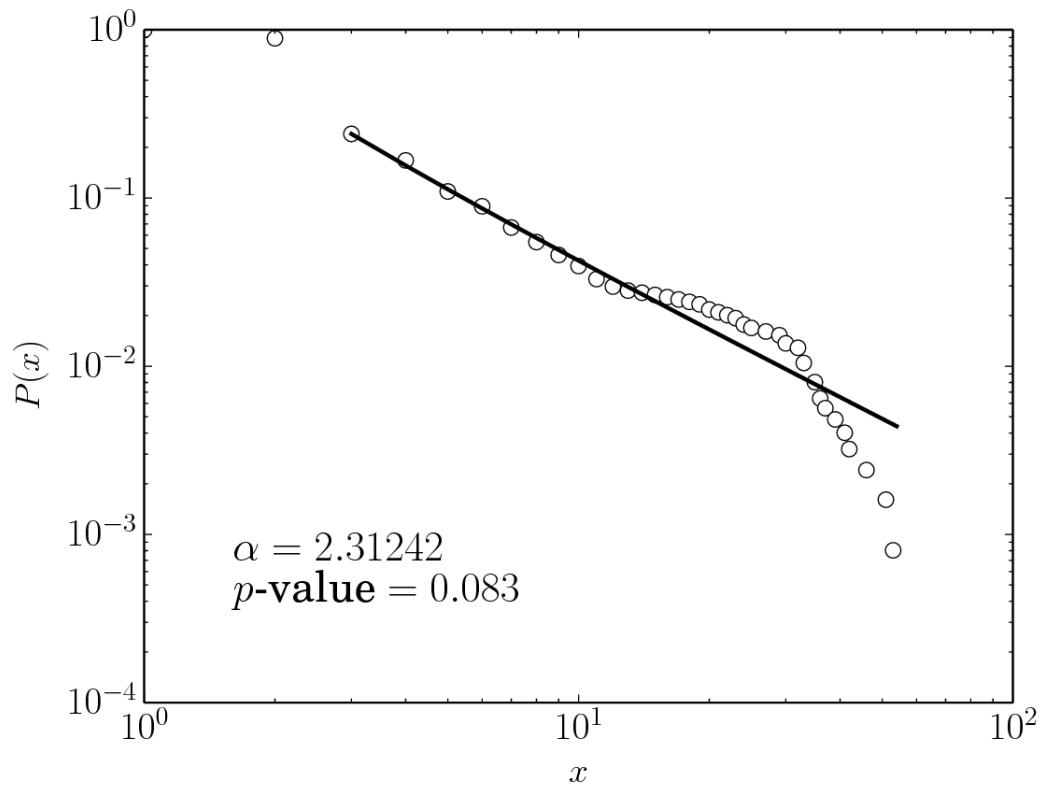


Figure 9: Beethoven - Sonata No. 11 in Bb major, Opus 22 (1800)

→ Scale-free compatibility: NO (p -value < 0.10)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

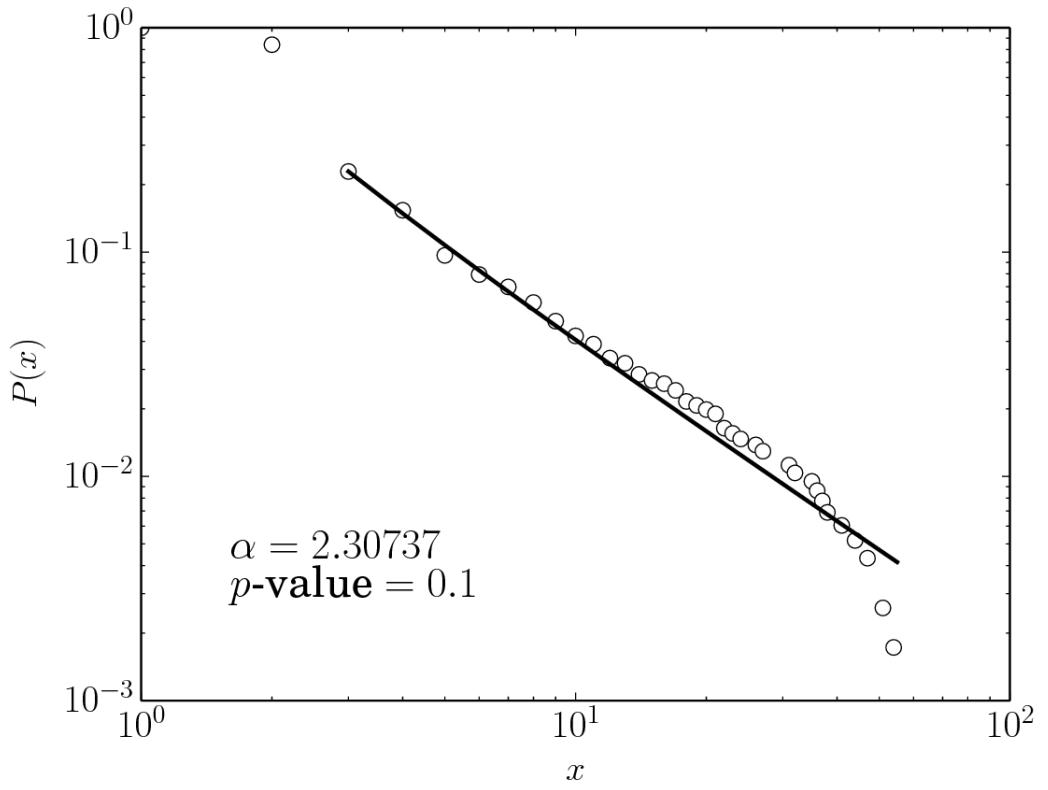


Figure 10: Beethoven - Sonata No. 14 in C# minor (Moonlight), Opus 27/2 (1801)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: NO (MSPL outside the confidence interval)

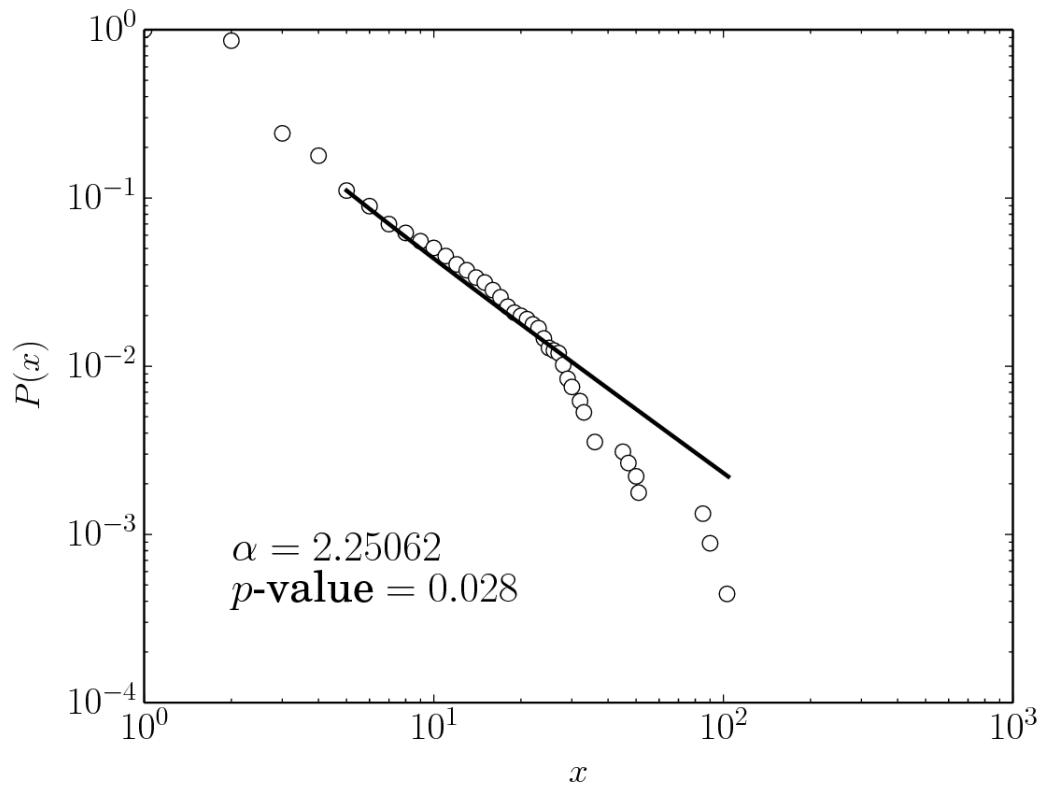


Figure 11: Beethoven - Sonata No. 21 in C major (Waldstein), Opus 53 (1804)

→ Scale-free compatibility: NO (p -value < 0.10)

→ Small-world compatibility: NO (MSPL outside the confidence interval)

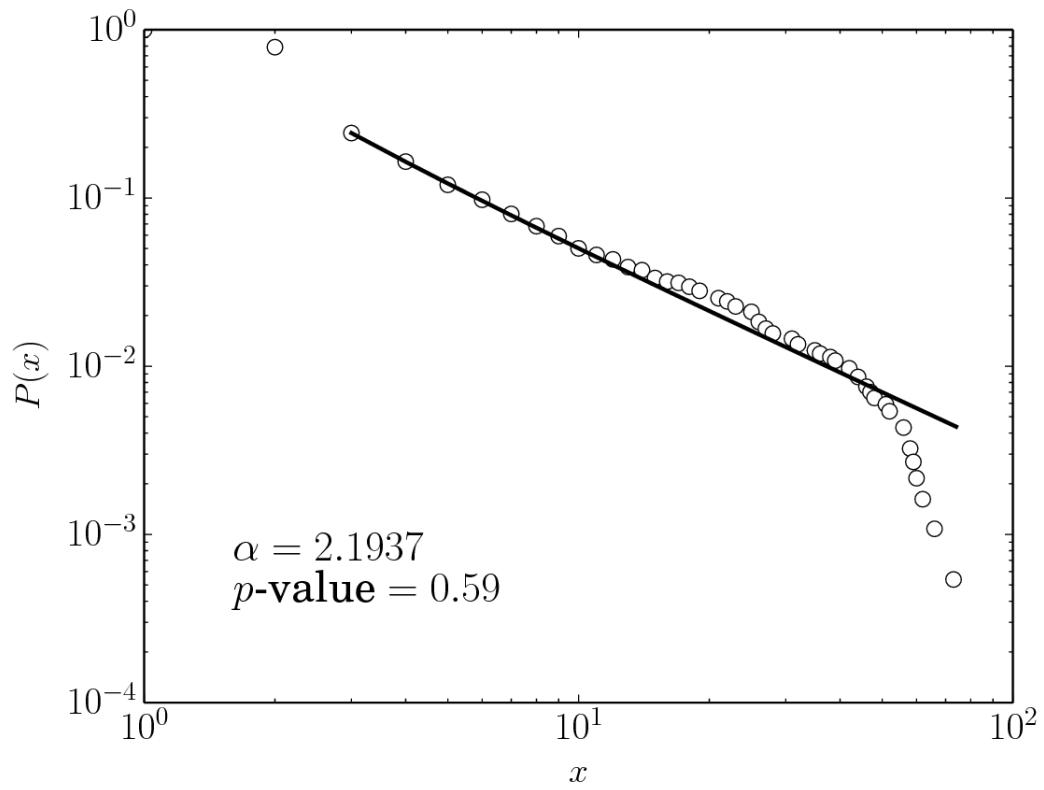


Figure 12: Beethoven - Sonata No. 23 in F minor (Appassionata), Opus 57 (1804)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

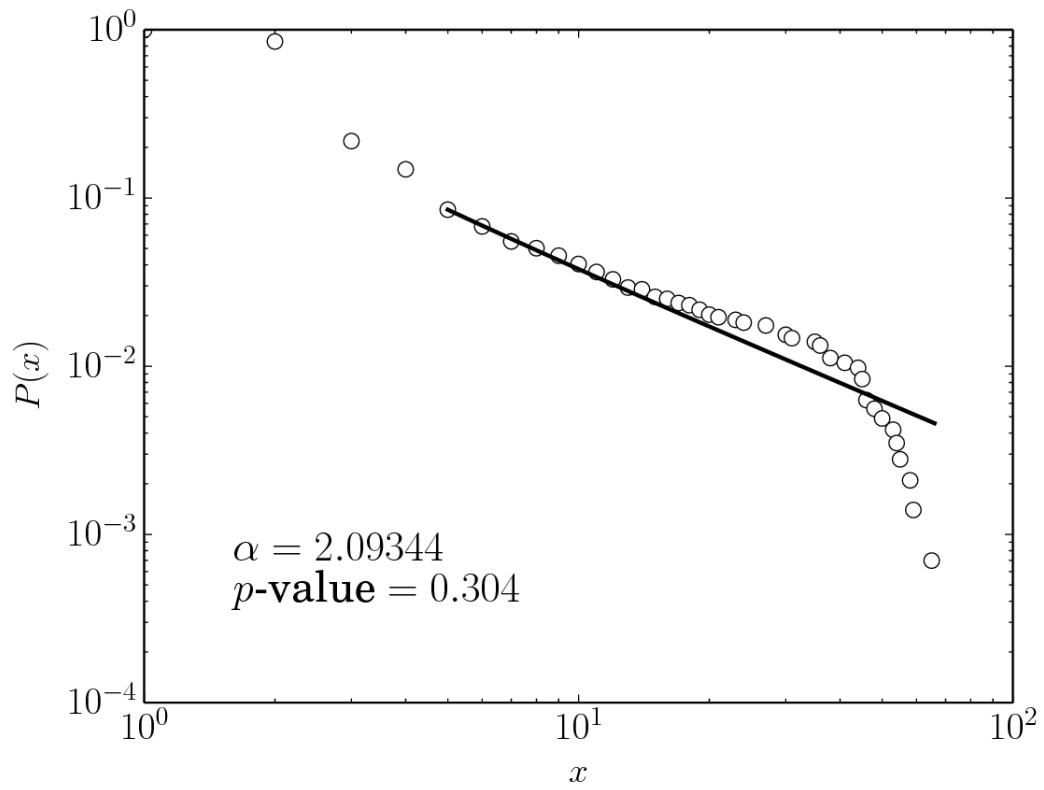


Figure 13: Beethoven - Sonata No. 26 in Eb major (Les Adieux), Opus 81a (1809)

→ Scale-free compatibility: YES (exponent α – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

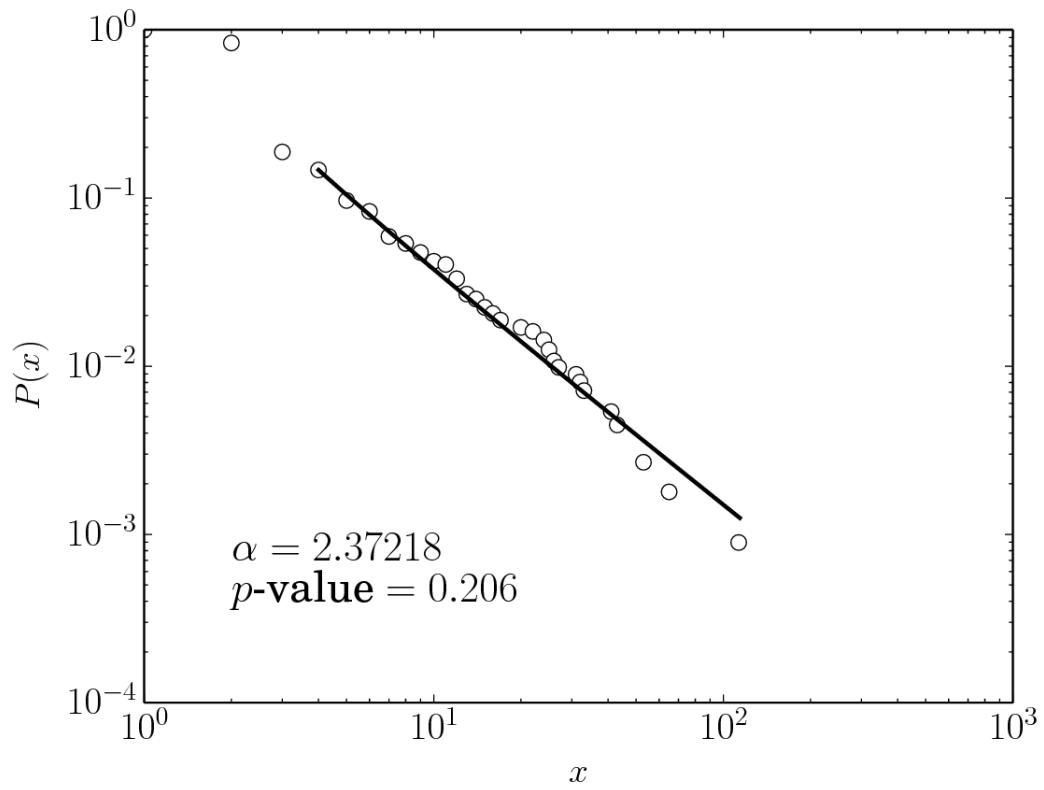


Figure 14: Beethoven - Sonata No. 27 in E minor, Opus 90 (1814)

→ Scale-free compatibility: YES (exponent α – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: NO (MSPL outside the confidence interval)

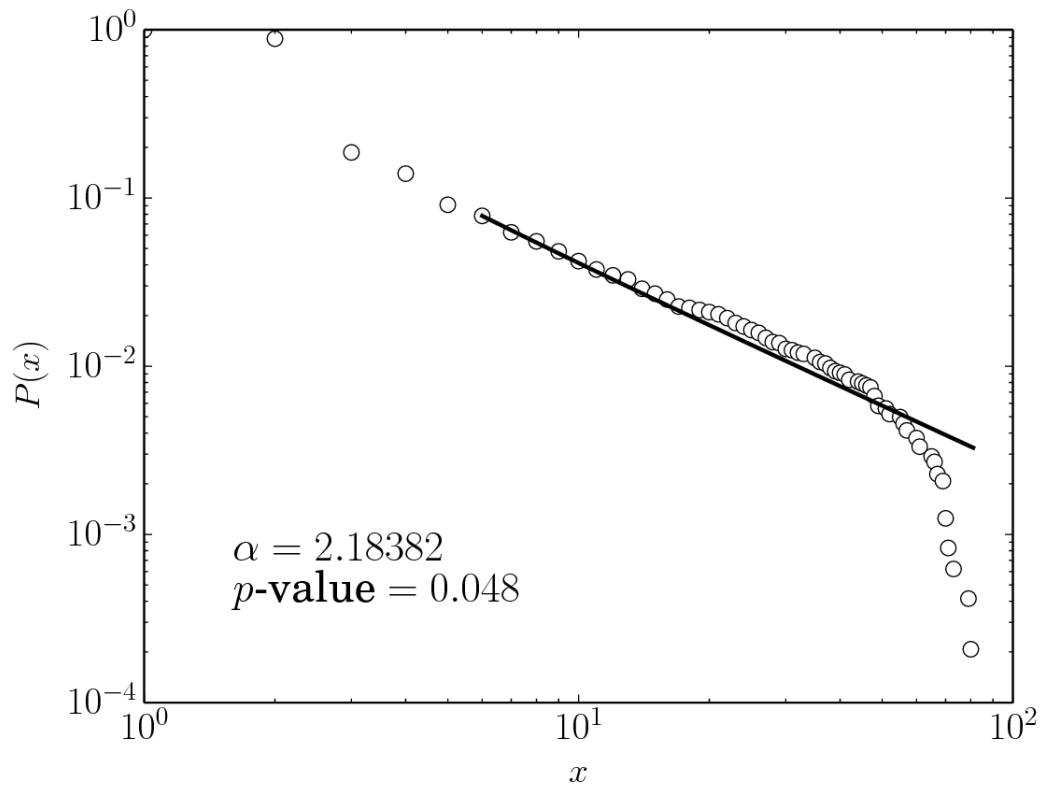


Figure 15: Beethoven - Sonata No. 29 in Bb major (Hammerklavier), Opus 106 (1818)

→ Scale-free compatibility: NO (p -value trend to stretched exponential)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

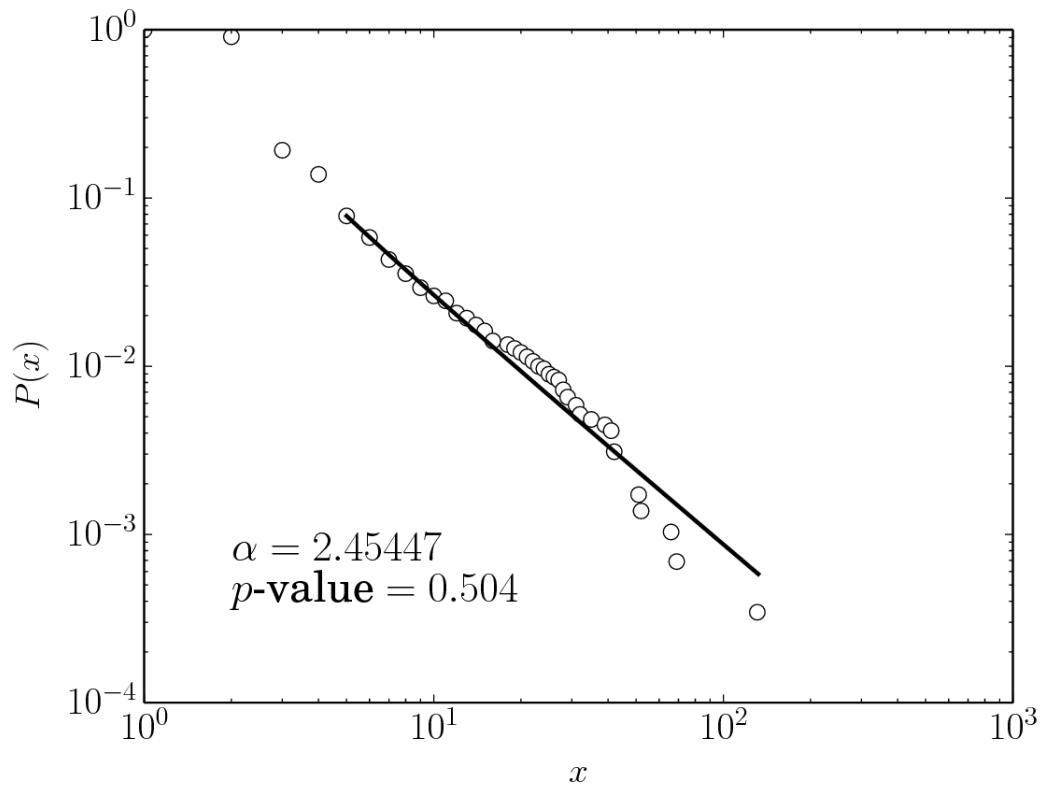


Figure 16: Brahms - Sonata in C major, Opus 1 (1853)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: NO (MSPL outside the confidence interval)

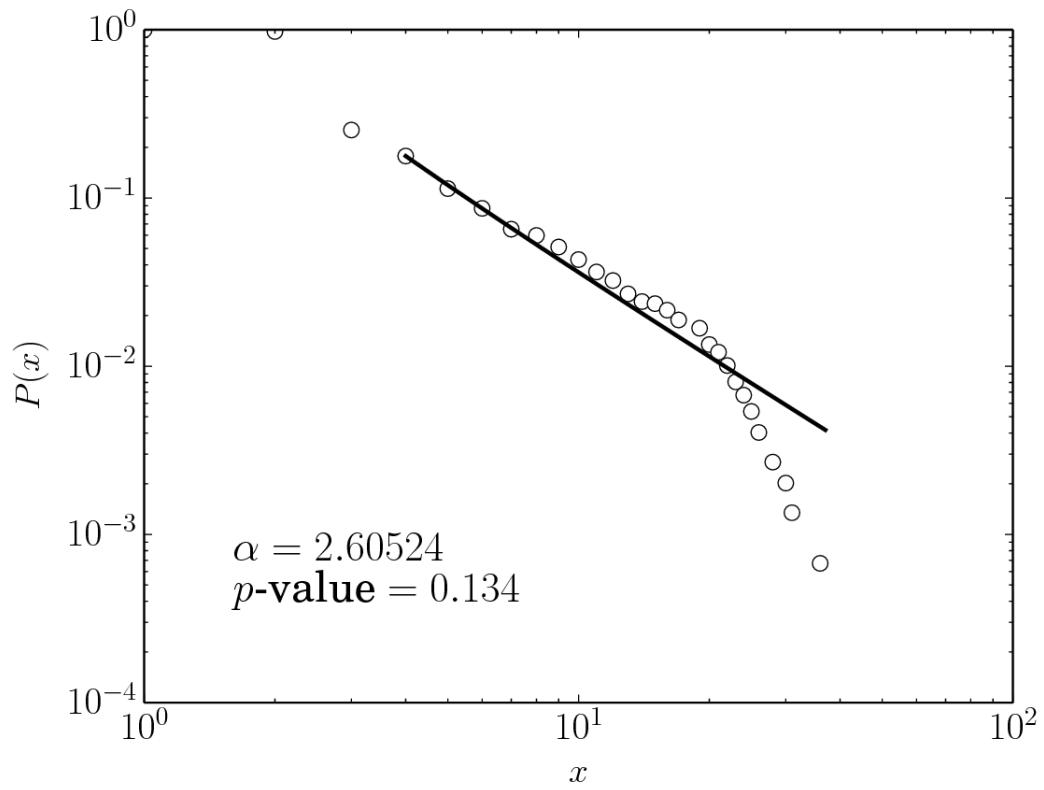


Figure 17: Chopin - Piano sonata No. 2 in Bb minor, Opus 35 (1839)

- Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)
- Small-world compatibility: NO (MSPL outside the confidence interval)

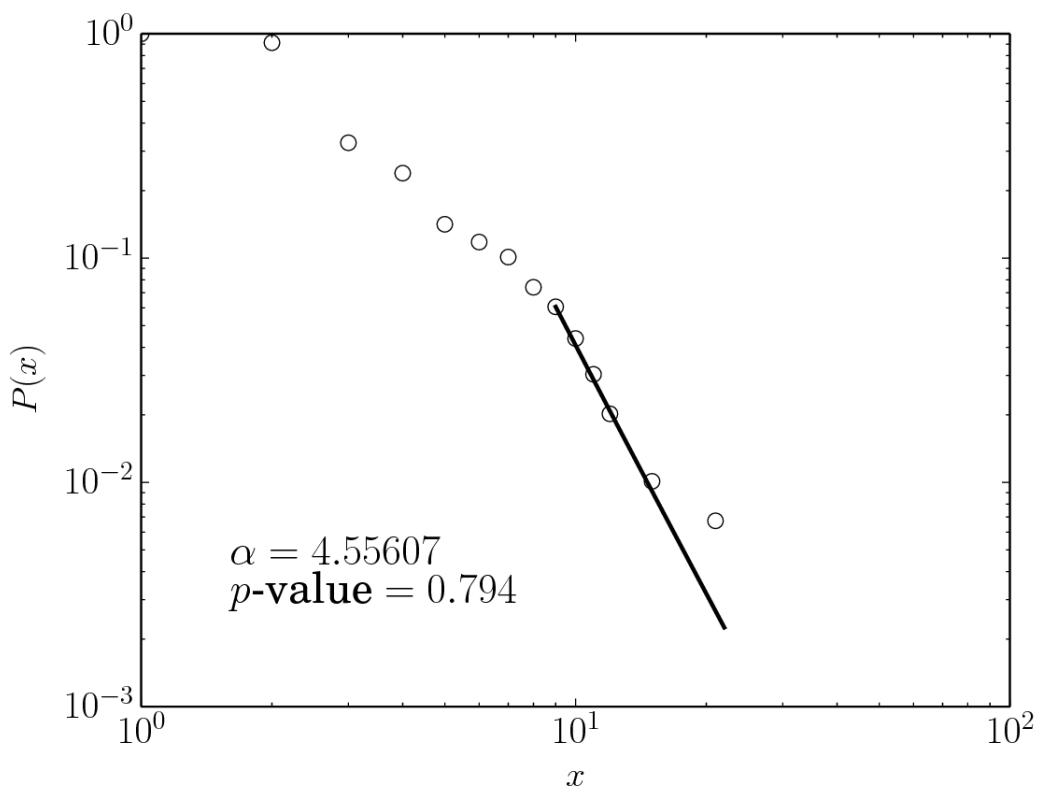


Figure 18: Clementi - Sonatina Opus 36 No. 1 in C major, Opus 36 (1797)

-> Scale-free compatibility: NO (exponent $a > 3$)

-> Small-world compatibility: NO (MSPL outside the confidence interval)

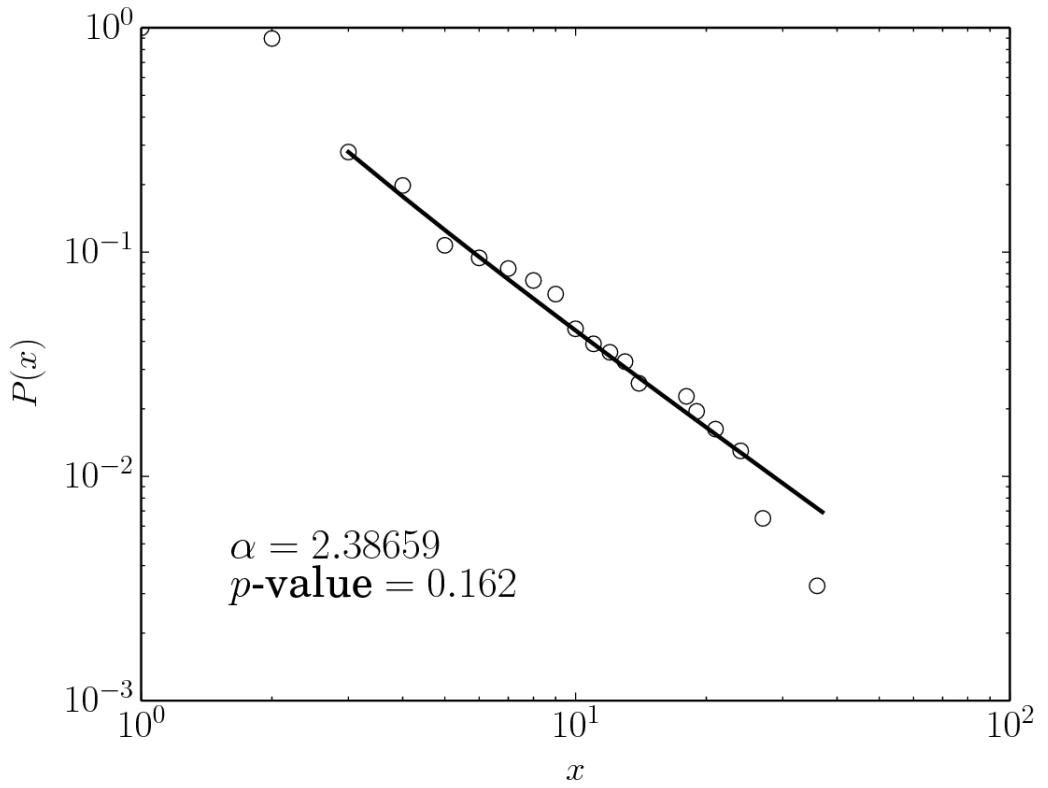


Figure 19: Clementi - Sonatina Opus 36 No. 2 in G major, Opus 36 (1797)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

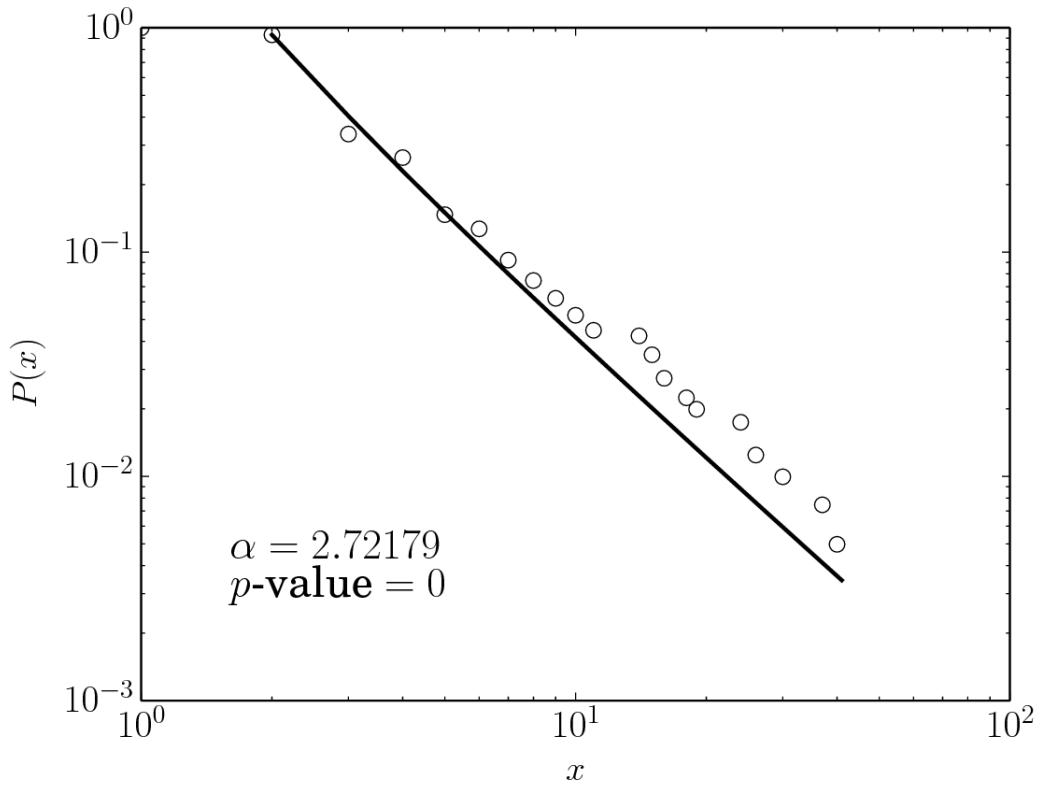


Figure 20: Clementi - Sonatina Opus 36 No. 3 in C major, Opus 36 (1797)

-> Scale-free compatibility: NO (p -value < 0.10)

-> Small-world compatibility: NO (MSPL outside the confidence interval)

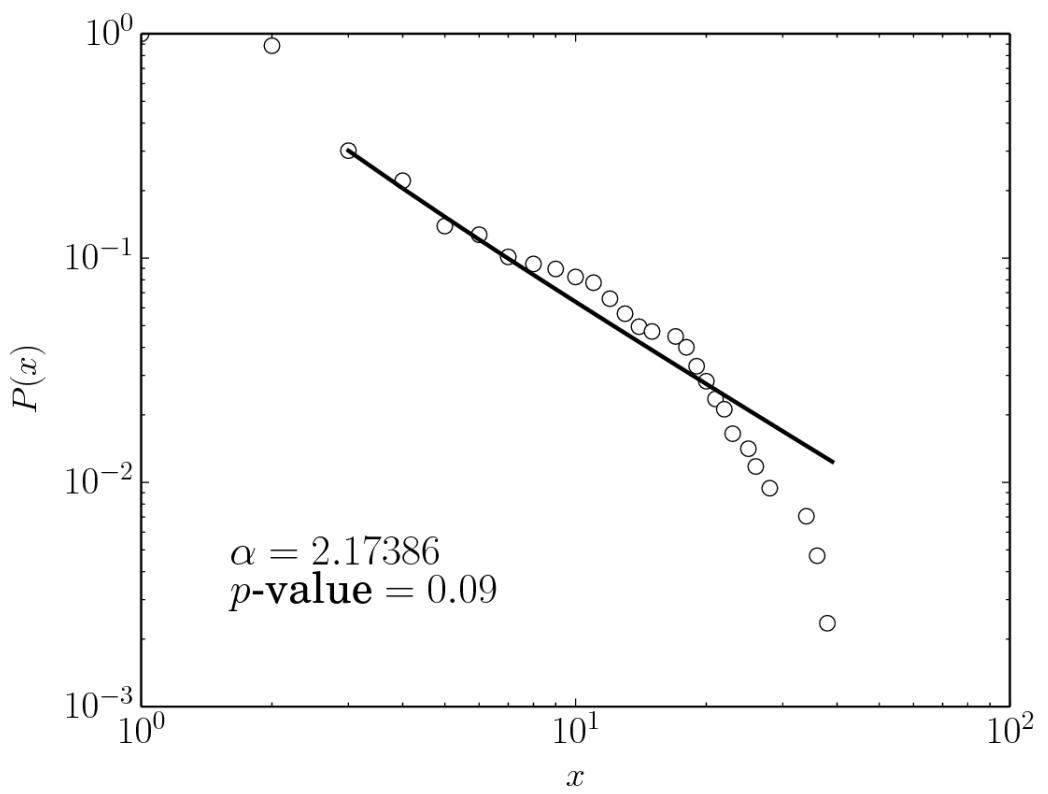


Figure 21: Clementi - Sonatina Opus 36 No. 4 in F major, Opus 36 (1797)

-> Scale-free compatibility: NO (p -value trend to exponential)

-> Small-world compatibility: NO (MSPL outside the confidence interval)

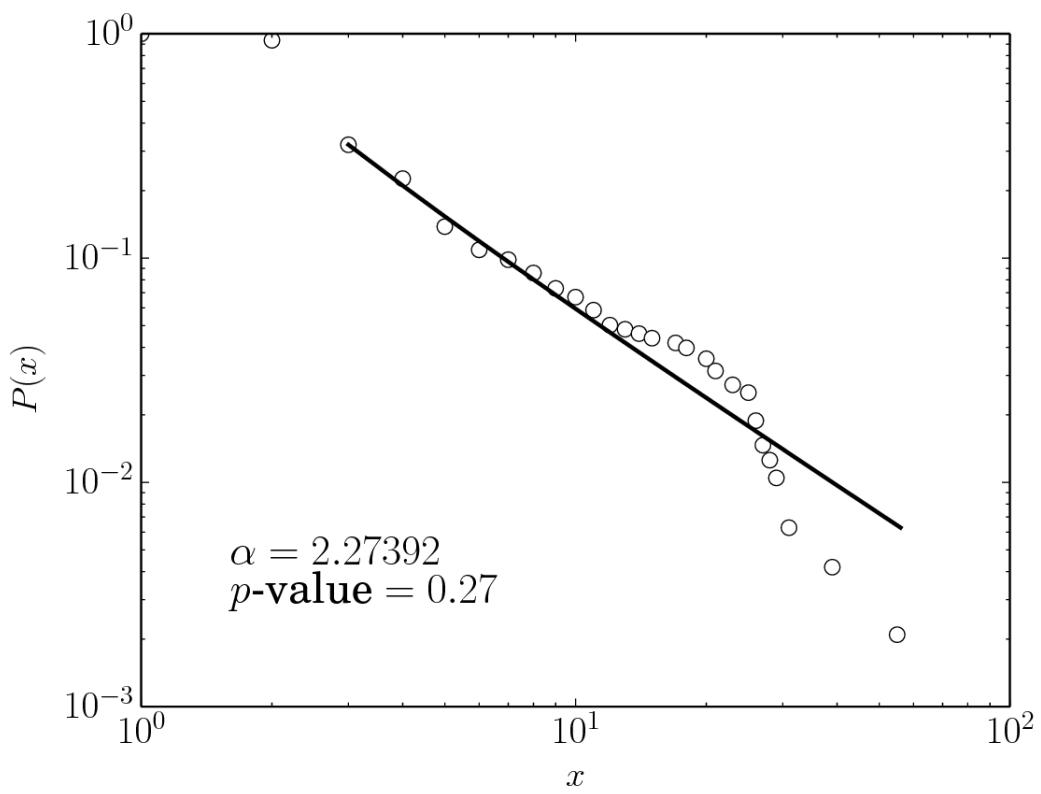


Figure 22: Clementi - Sonatina Opus 36 No. 5 in C major, Opus 36 (1797)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: NO (MSPL outside the confidence interval)

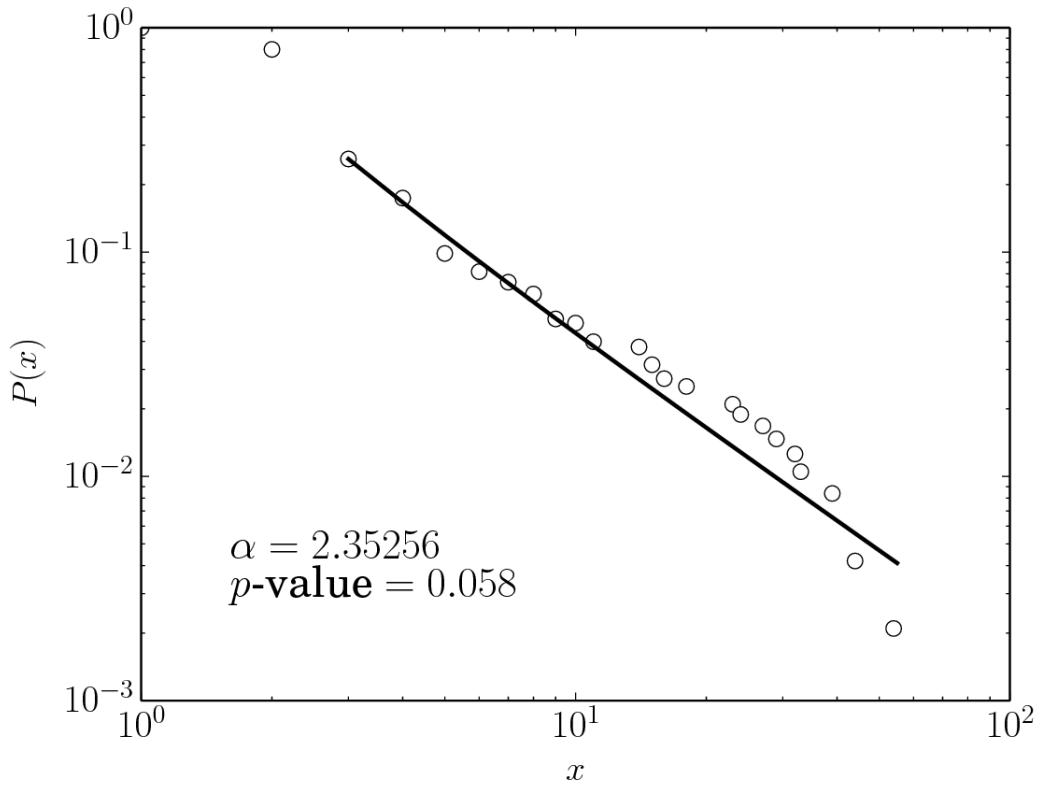


Figure 23: Clementi - Sonatina Opus 36 No. 6 in D major, Opus 36 (1797)

→ Scale-free compatibility: NO (p -value < 0.10)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

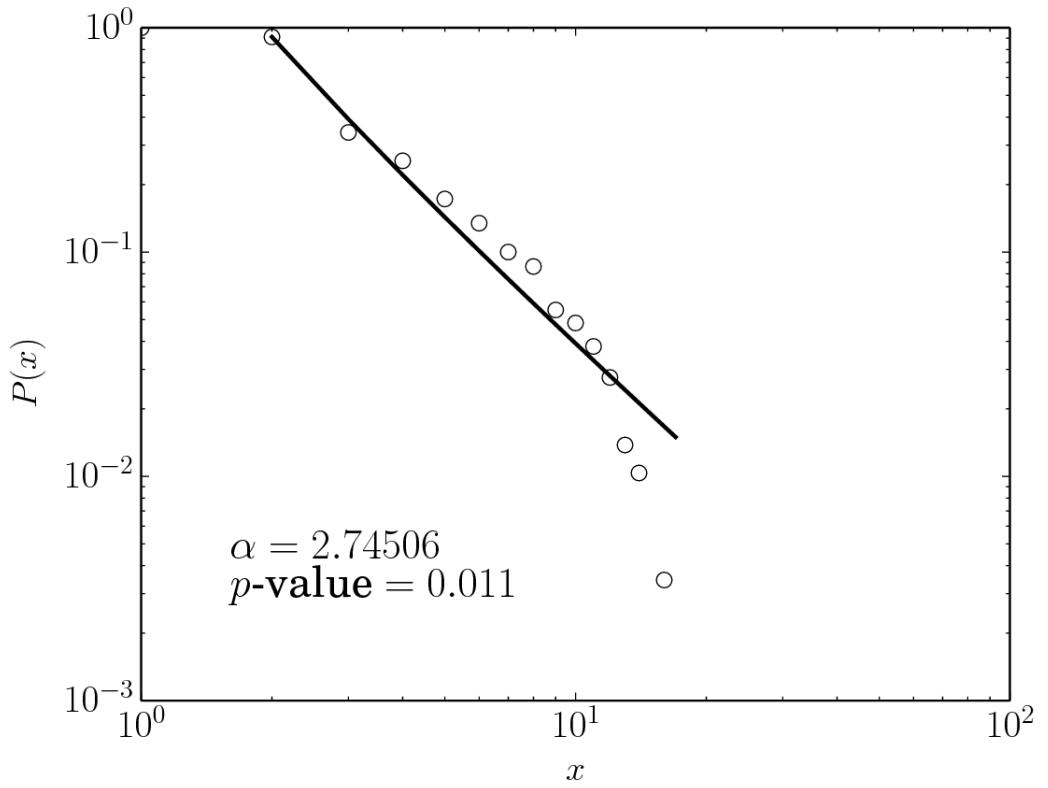


Figure 24: Haydn - Piano Sonata in F major, Hoboken XVI:9 (1766)

→ Scale-free compatibility: NO (p -value < 0.10 and p -value trend to stretched exponential)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

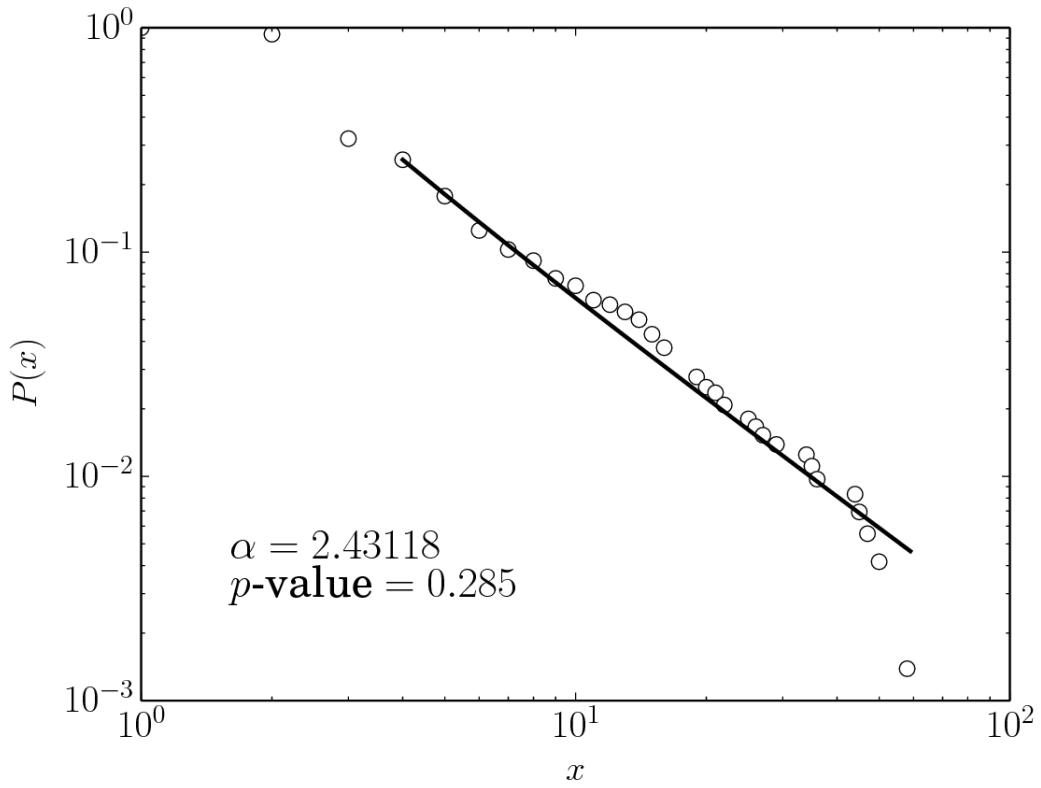


Figure 25: Haydn - Piano Sonata in D major, Hoboken XVI:33 (1778)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

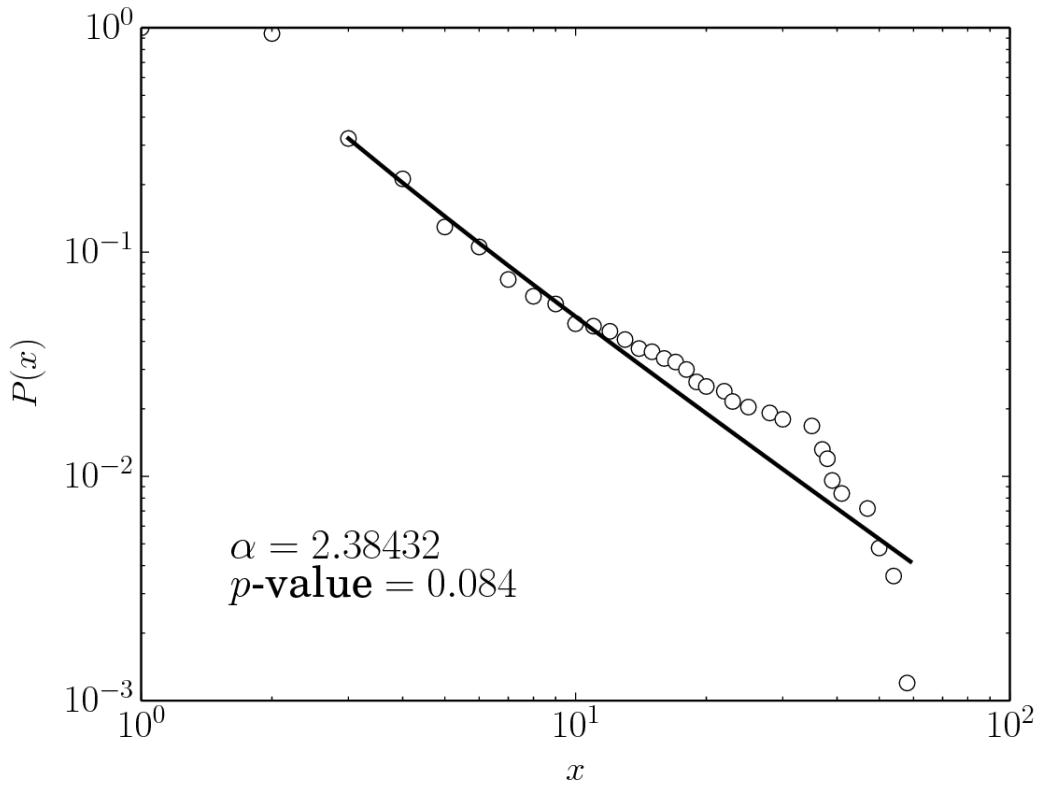


Figure 26: Haydn - Piano Sonata in C major, Hoboken XVI:35 (1780)

→ Scale-free compatibility: NO (p -value < 0.10)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

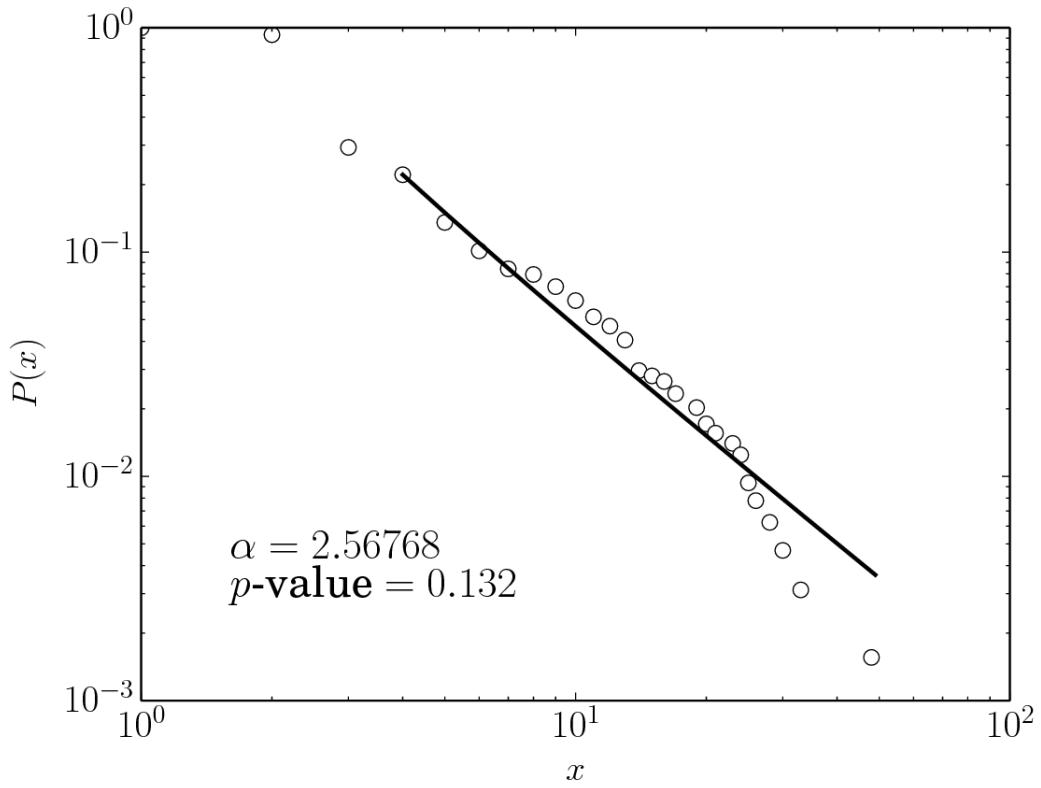


Figure 27: Haydn - Piano Sonata in G major, Hoboken XVI:40 (1784)

→ Scale-free compatibility: NO (p -value trend to stretched exponential)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

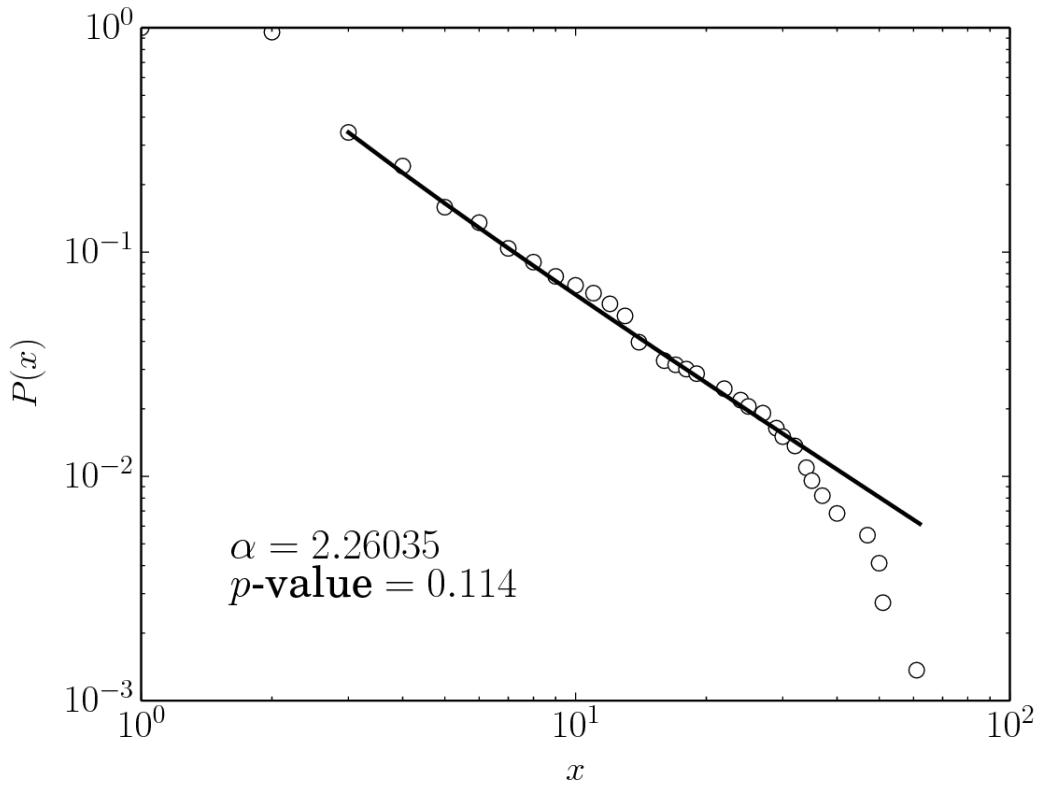


Figure 28: Haydn - Piano Sonata in Ab major, Hoboken XVI:43 (1783)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

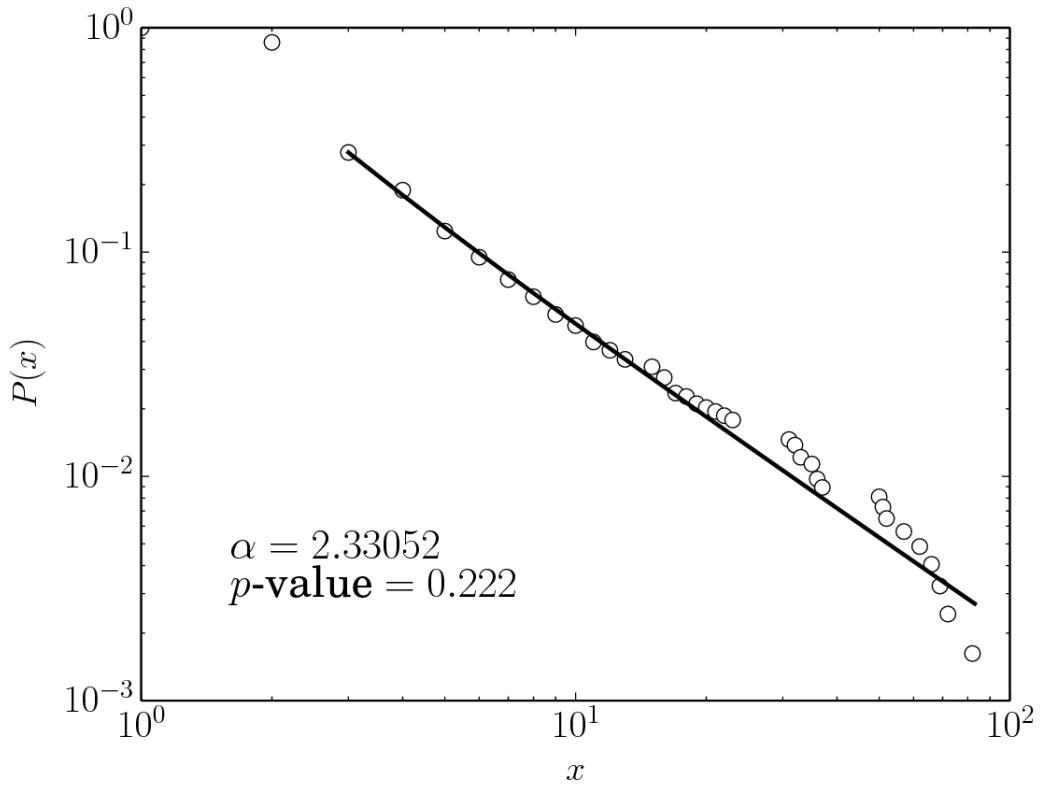


Figure 29: Mozart - Sonata No. 8 in D major, KV 311 (1777)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

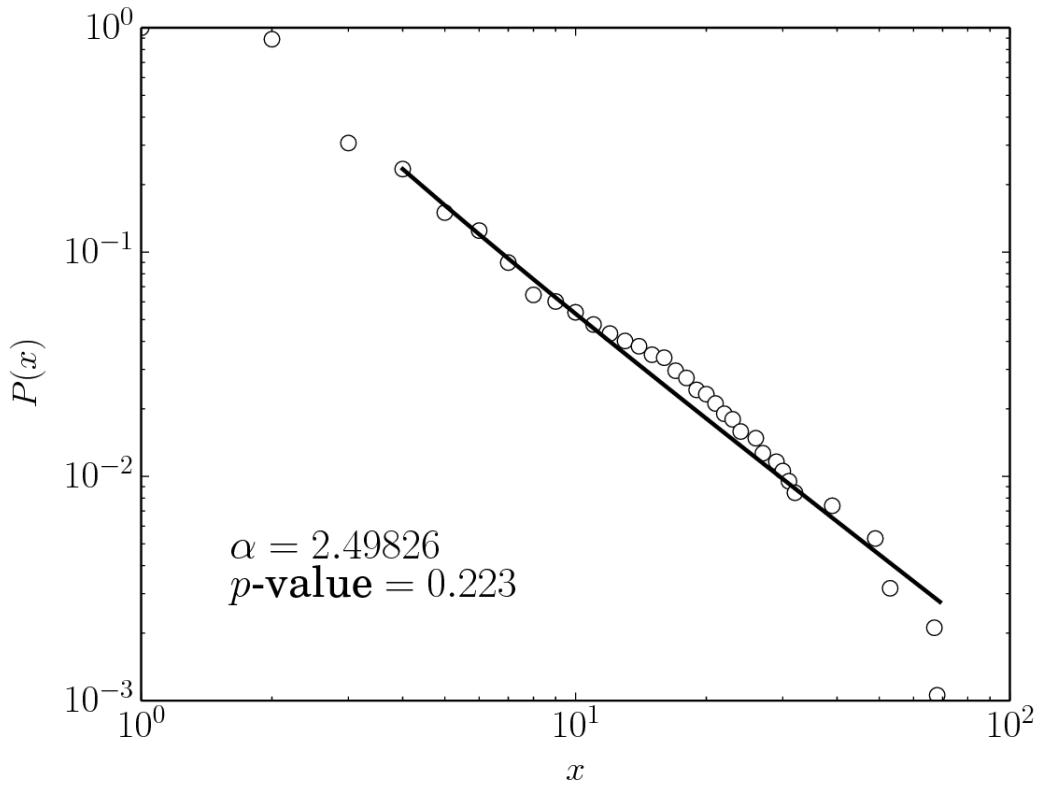


Figure 30: Mozart - Sonata No. 10 in C major, KV 330 (1783)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

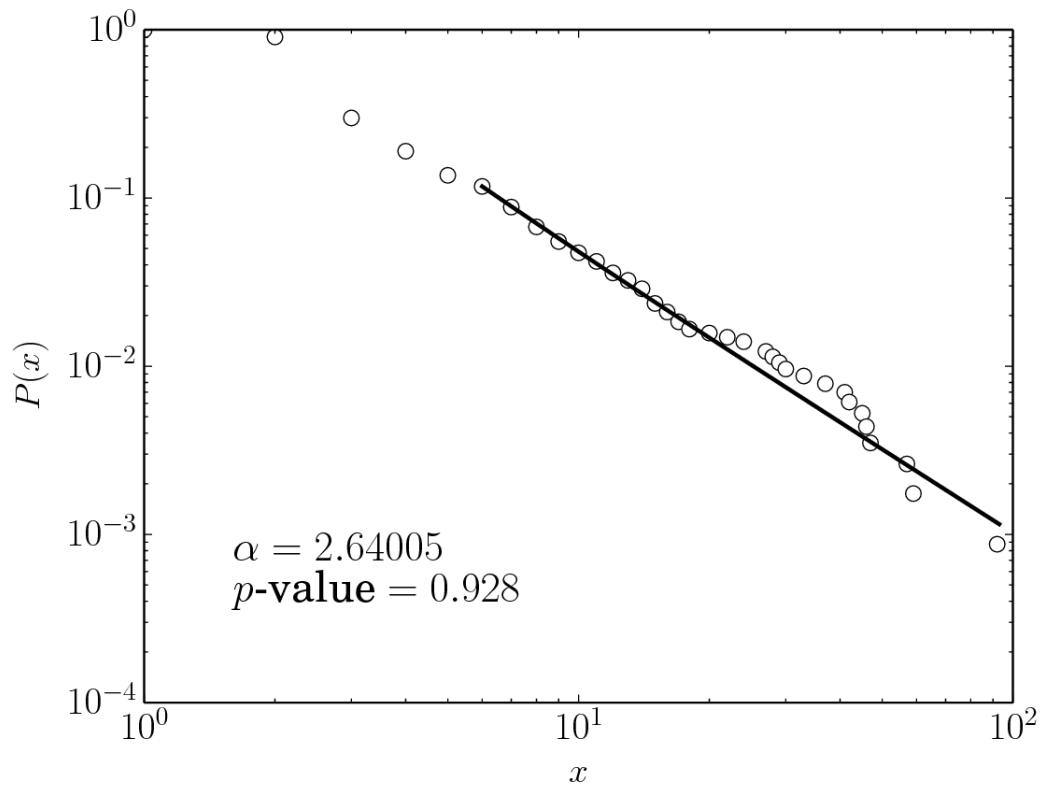


Figure 31: Mozart - Sonata No. 11 in A major (Alla Turca), KV 331 (1783)

→ Scale-free compatibility: YES (exponent α – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

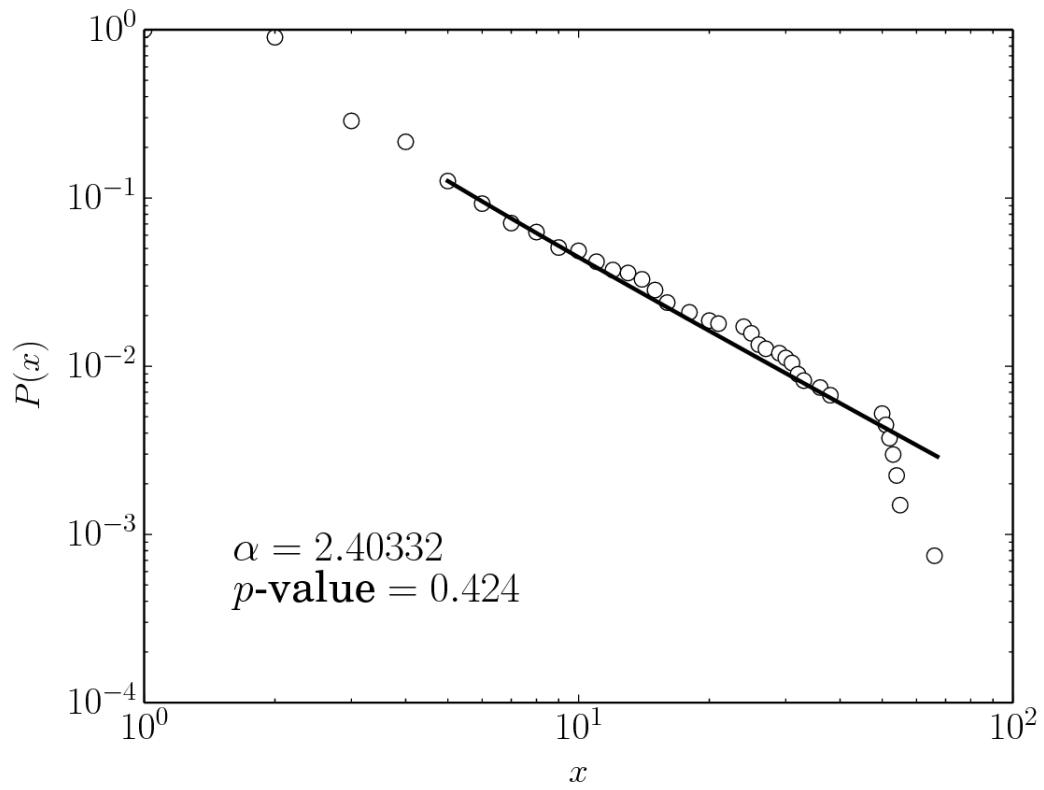


Figure 32: Mozart - Sonata No. 12 in F major, KV 332 (1783)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

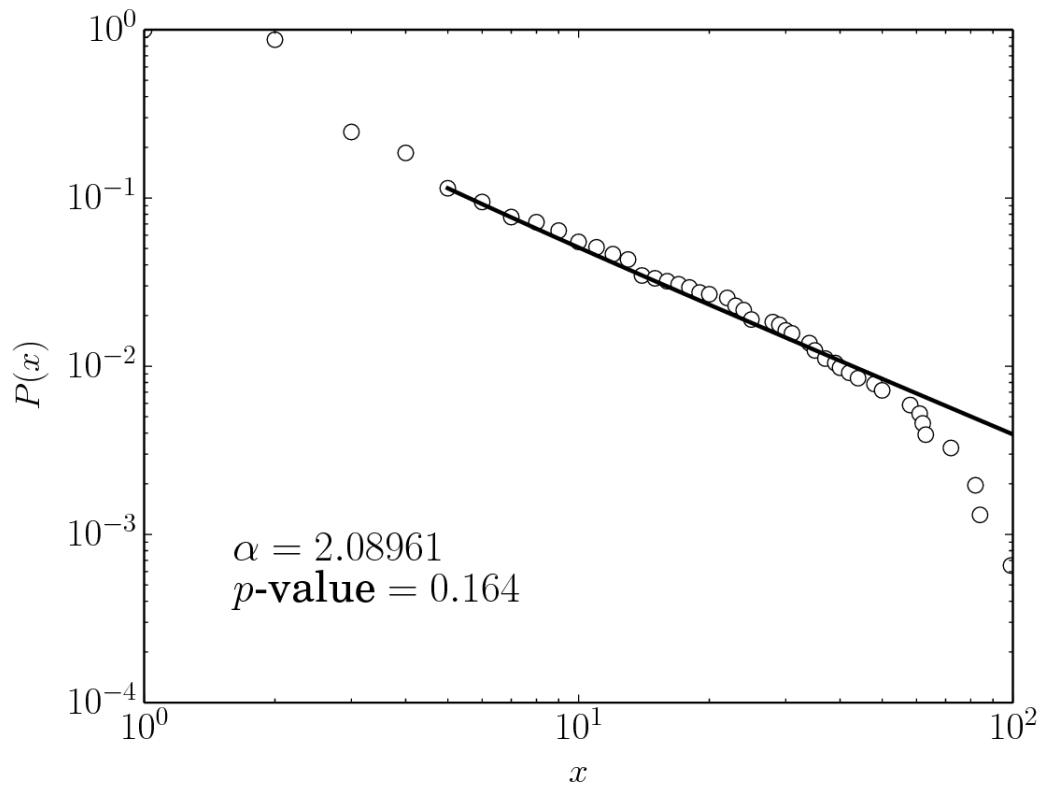


Figure 33: Mozart - Sonata No. 13 in Bb major, KV 333 (1783)

- Scale-free compatibility: NO (p -value trend to stretched exponential)
- Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

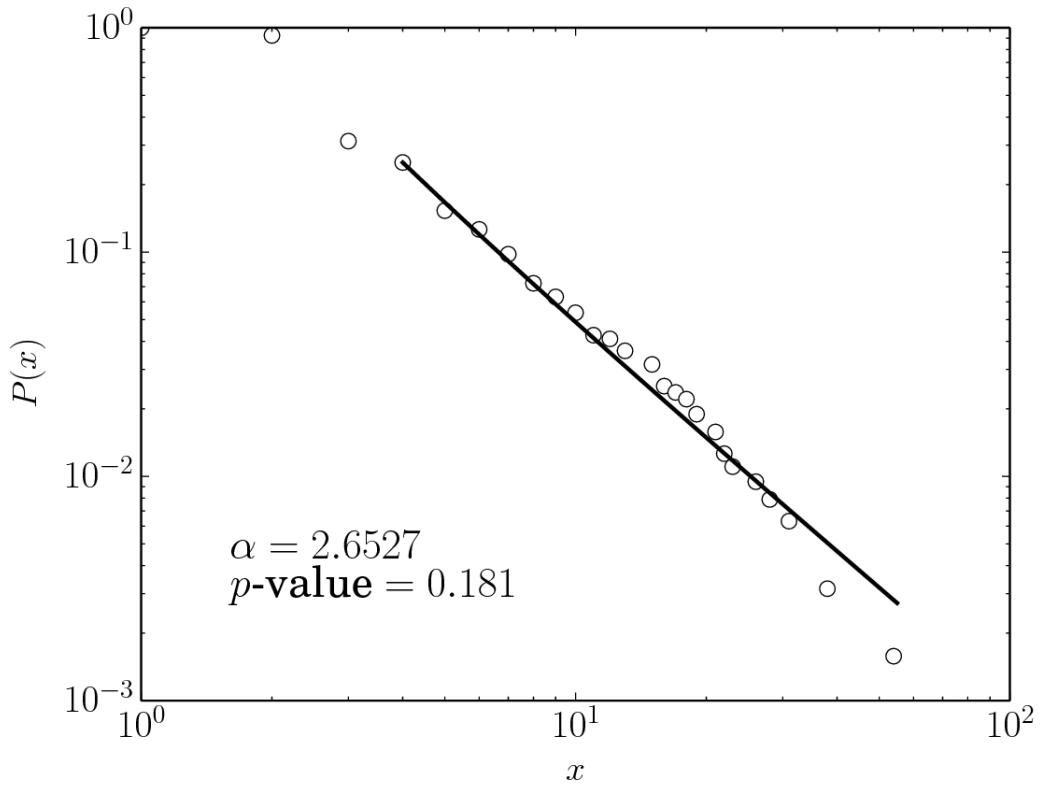


Figure 34: Mozart - Sonata No. 16 in C major (Sonata facile), KV 545 (1788)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

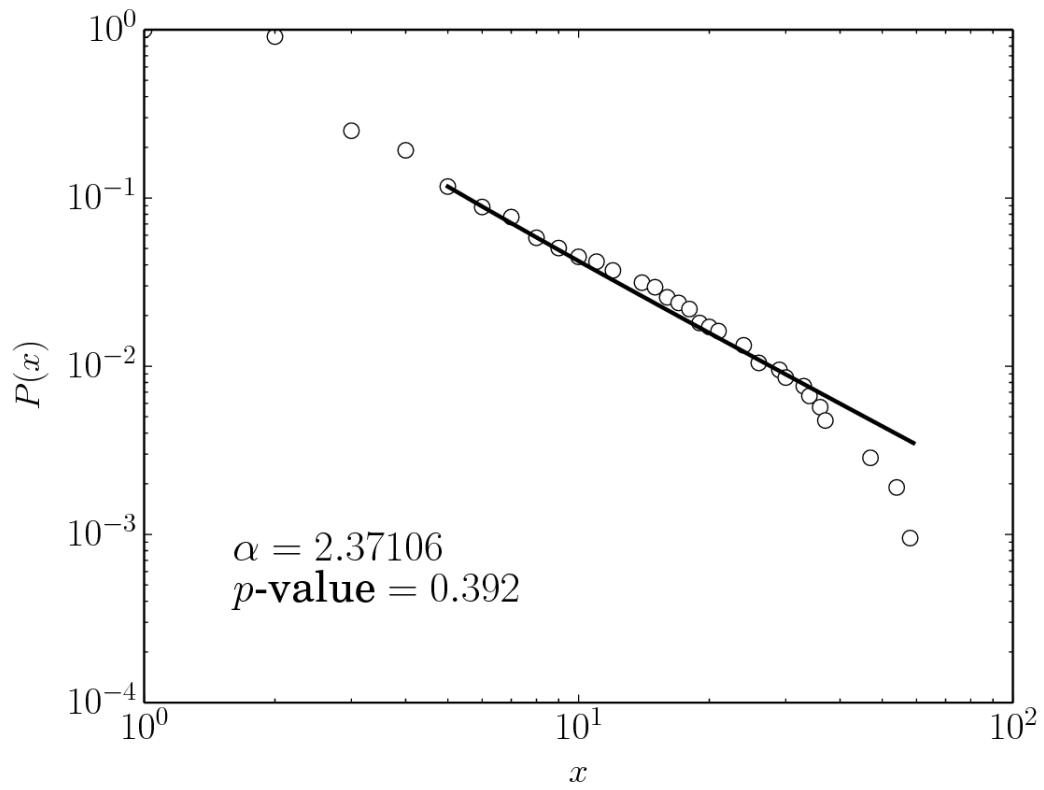


Figure 35: Mozart - Sonata No. 17 in Bb major, KV 570 (1789)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

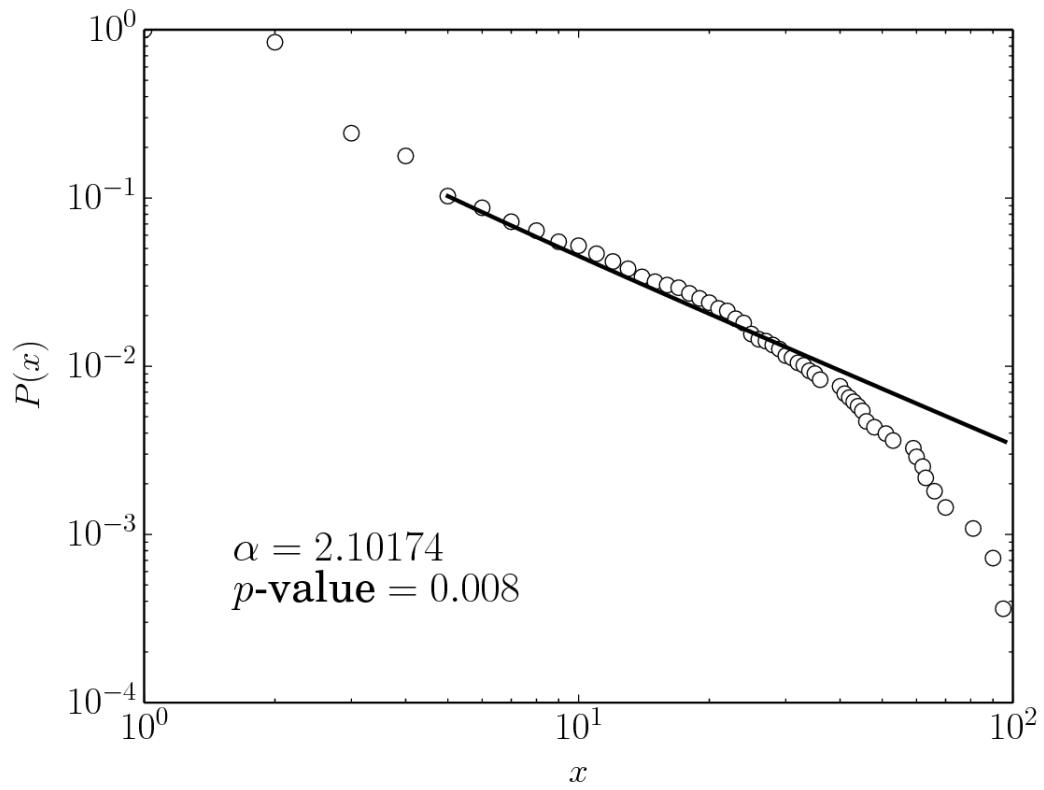


Figure 36: Schubert - Piano Sonata in Bb major, D 960 (1828)

→ Scale-free compatibility: NO (p -value < 0.10 and p -value trend to stretched exponential)

→ Small-world compatibility: YES (MSPL inside the confidence interval, high ACC compared to random and small-world networks)

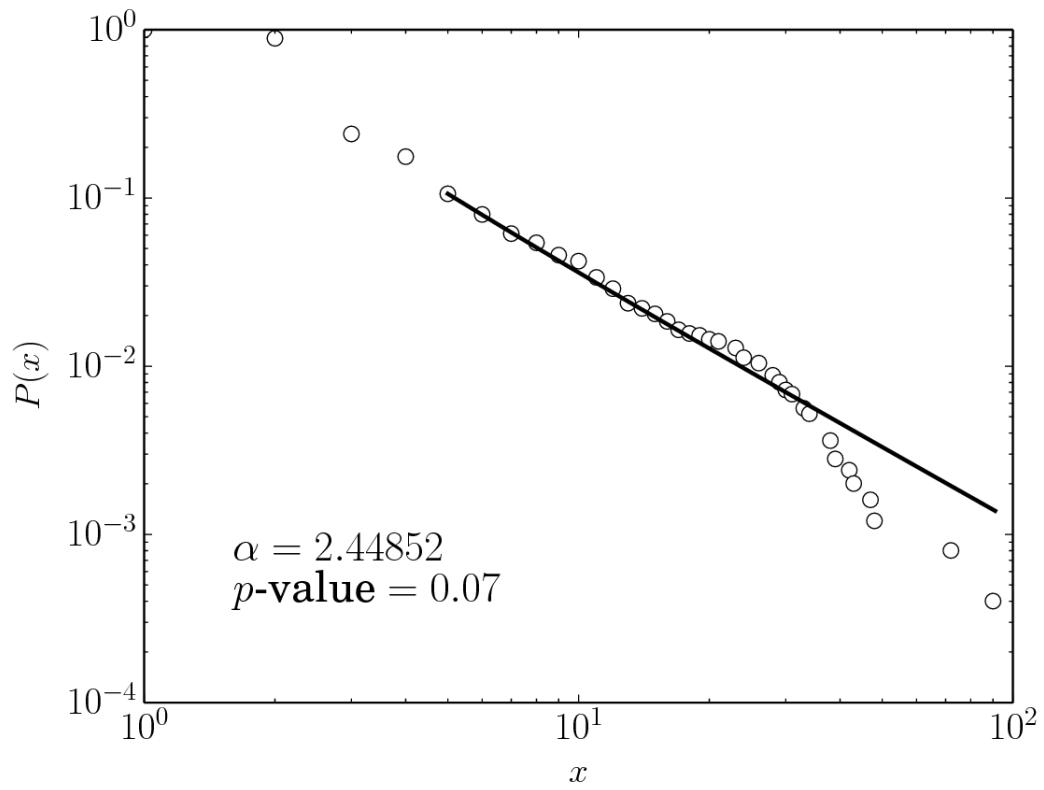


Figure 37: Schubert - Fantasia in C major (Wanderer), D 760, Opus 15 (1822)

→ Scale-free compatibility: NO (p -value < 0.10 and p -value trend to stretched exponential)

→ Small-world compatibility: NO (MSPL outside the confidence interval)

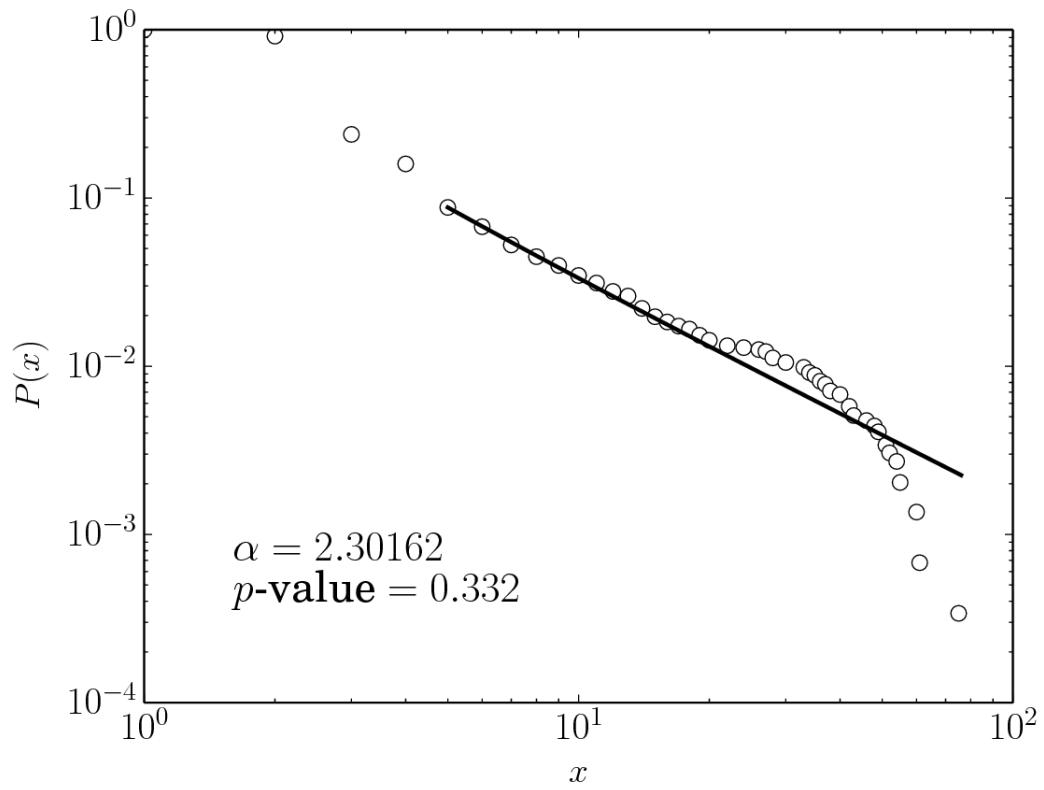


Figure 38: Schubert - Piano Sonata in D major, D 850, Opus 53 (1825)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: NO (MSPL outside the confidence interval)

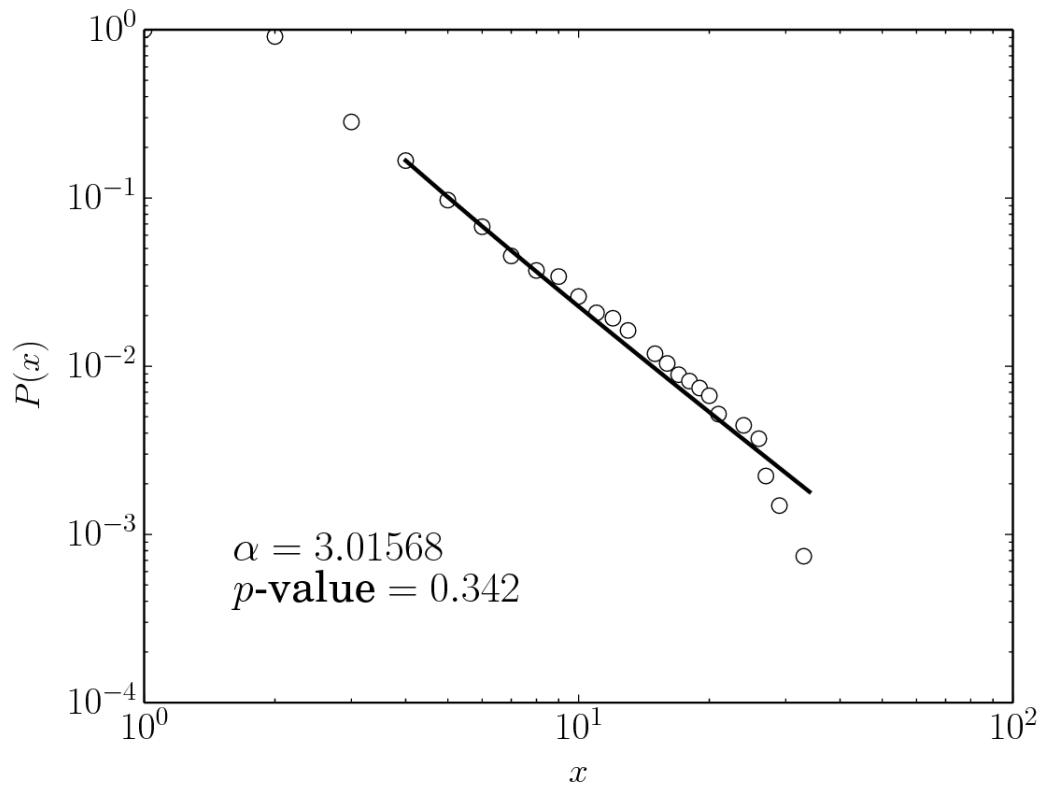


Figure 39: Schubert - Piano Sonata in A minor, D 784, Opus 143 (1823)

→ Scale-free compatibility: YES (exponent a – OK, p -value – OK, likelihood tests – OK)

→ Small-world compatibility: NO (MSPL outside the confidence interval)

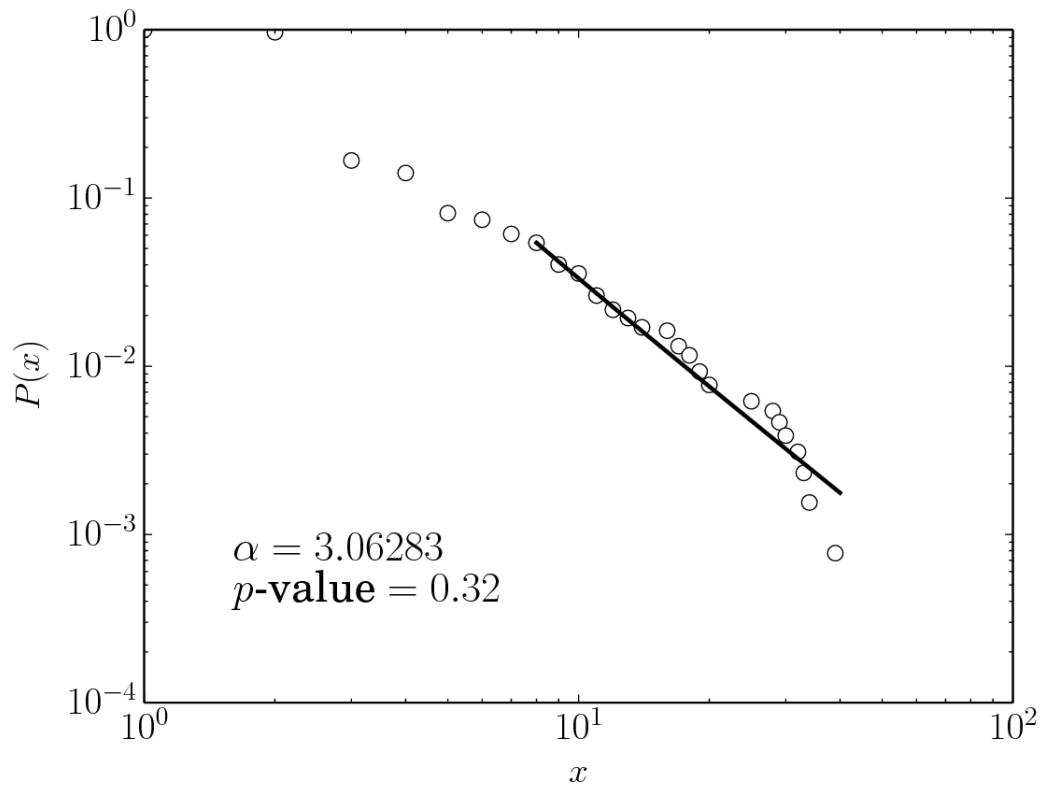


Figure 40: Shostakovich - Piano Quintet in G minor, Opus 57 (1940)

- > Scale-free compatibility: NO (exponent a on the edge and p -value trend to log-normal)
- > Small-world compatibility: NO (MSPL outside the confidence interval)