**On volumes of arithmetic locally symmetric spaces**

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Let $G$ be a semi–simple algebraic group defined over a number field $k$, $K \subset G_{\mathbb{R}}$ – a maximal compact subgroup, $\Gamma \subset G(k)$ – an arithmetic subgroup of $G$. We are interested in computing the volumes of the locally symmetric spaces

$$\mu(\Gamma \backslash G_{\mathbb{R}}/K)$$

with respect to a naturally normalized Haar measure $\mu$ on the group $G$. This problem has a long history which goes back to the work of Minkowski and Siegel.

There are two approaches which can give closed formulas for the volumes in a general situation. One is to use Eisenstein series, concerning this we would like to mention the papers of G. Shimura from 1997 – 1999. The second approach uses the Tamagawa measure on $G(\mathbb{A})$ and Bruhat–Tits theory. A recent breakthrough in this direction is due to G. Prasad who gave a closed formula for the covolume of a principal arithmetic subgroup of $G$ in his article in Publ. IHES, 1989.

The aim of my lecture is to discuss the second approach and to show how to use Prasad's volume formula for the actual computations. As an application we present the following result:

**Theorem (B).** *For any $n = 2r \geq 4$ there exists a* unique *compact orientable arithmetic hyperbolic $n$–orbifold $O^n_{min}$ of the smallest volume. It is defined over $k = \mathbb{Q}[\sqrt{5}]$ and has Euler characteristic*

$$|\chi(O^n_{min})| = \frac{\lambda(r)}{N(r)4^{r-1}} \prod_{i=i}^{r} |\zeta_k(1-2i)|$$

*where $\zeta_k$ is the Dedekind zeta function of $k$, $N(r) \in \mathbb{Z}$ and $\lambda(r) \in \mathbb{Q}$ are constants such that $1 \leq N(r) \leq 4$, $\lambda(r) = 1$ for even $r$ and $1 \leq N(r) \leq 8$, $\lambda(r) = 2^{-1}(4^r - 1)$ for $r$ odd.*

The methods of Bruhat–Tits theory and results of G. Prasad appear to be effective in proving this theorem. We do not know how to get it using the first approach. Considering this, it could be interesting to see how far one can proceed with this method. In particular, we would like to ask whether it is possible to apply it to the problems of counting integer points on the *affine* symmetric spaces.