24/06/2021 Jacobi forms of arbitrary index.

Let T be the moduli space constructed in GMCD-JFO. Its regular functions are interpreted as Jacobi forms of index zero.

Def: A Jacobi form of index (yet to be determined) is any global multivalued function F on T such that

$$R_z \cdot \ln F = A \in \mathcal{O}_T \quad (*)$$
$$R_\tau \cdot \ln F = B \in \mathcal{O}_T$$

where $\mathcal{O}_T$ is the ring of global regular functions on T

Prop: For $A, B \in \mathcal{O}_T$ such an F exist if and only if $R_\tau A = R_z B$

Proof: $\Rightarrow$ is just derivating of and the fact that R_z, R_τ commute.

$\Leftarrow$

For this we note that R_z, R_τ do not have singularities in T and so $[R_z, R_\tau] = 0$ implies that in a coordinate system $(z_1, z_2, \dots)$ of any point of T we can write

$$R_\tau = \frac{\partial}{\partial z_1} \qquad R_z = \frac{\partial}{\partial z_2}$$

the rest of the proof is easy. ■

In particular, F has analytic continuation to every where in T. Fix F in a small neigh of b in T. We get

$$\chi : \pi_1(T, b) \longrightarrow \mathbb{C}^* \text{ a character s.t.}$$

analytic continuation of F along $\gamma \in \pi_1(T, b)$ is $F \cdot \chi(\gamma)$.