01/04/2020 Rio de Janeiro: Modular vector fields for K3

Conjecture: The moduli of (X, i, α_1), X a K3, $i: L \to NS(X)$ and $\alpha \in H^{3,0}(X)$, is always a quasi-affine variety.

Example: For Clingher-Doran family this moduli is

$$S =: \mathrm{Spec}(\mathbb{Q}[a,b,c,d,\tfrac{1}{\Delta}]) \setminus \{c=d=0\}$$

Thm: The cohomology bundle over S above is trivial

Proof: Notation of [DHMW]. We have to show that the 5 differential forms in Prop. 6.3, $\underbrace{\ }_{\omega_1,\omega_2,\omega_3,\omega_4,\omega_5}$ gives us the trivialization. we have computed the G.M connection in this basis: $\nabla\omega = A\omega$ $\omega = [\omega_1, \omega_2, \dots, \omega_5]^{tr}$. we compute it in $\bigwedge_{i=1}^{5} H^2_{dR}$.

$$\nabla \omega_1 \wedge \omega_2 \wedge \dots \wedge \omega_5 = a \cdot \omega_1 \wedge \omega_2 \wedge \dots \wedge \omega_5.$$

we verfiy that a has no zeros in S and so at each point of S, $\omega_1, \omega_2, \dots \omega_5$ form a basis of H^2_{dR}. ∎

Thm: The pieces of Hodge filtration of H^2_{dR} are trivial bundles over S.

Proof: $F^2 \subseteq F^1 \subseteq F^0$, F^2 is trivial by construction of S and F^0 is trivial by previous theorem. Is the following true:

$$\boxed{\text{subbundles of a trivial bundle over a quasi-affine variety are trivial.}}$$

If Yes, we are done.

Thm: T is a quasi-affine variety: Proof: Thms above

Let f be a tame polynomial with non-zero discriminant and M be its G.M. system. Let also $M_k \subseteq M$ containing elements of pole order $\leq k$ along $f=0$. Let also $x^\beta, \beta \in I$ be a basis of Milnor module V_f.

Thm: M_k is a free R-module with basis

$$\frac{x^\beta dx_1 \wedge dx_2 \wedge \cdots dx_{n+1}}{f^k}, \beta \in I \qquad (*)$$

Cor: The pieces of Hodge filtration over the affine variety $\mathrm{Spec}(R)$ are trivial bundles. The trivialization of F^{n+1-k} is given by $(*)$ with $A_\beta < k$.

The proof of the above Thm must be similar to the same prove for $M_1 = H''$ and $M_0 = H'$.