Gaps of Fourier Quasicrystals and Lee-Yang Polynomials

The concept of "quasi-periodic" sets, functions, and measures is prevalent in diverse mathematical fields such as Mathematical Physics, Fourier Analysis, and Number Theory. The Poisson summation formula provides a "Fourier characterization" for discrete periodic sets, saying that the Fourier transform of the counting measure of a discrete periodic set is also a counting measure of a discrete periodic set. Fourier Quasicrystals (FQ) generalize this notion of periodicity: a counting measure of a discrete set is called a Fourier quasicrystal (FQ) if its Fourier transform is also a discrete atomic measure, together with some growth condition.

Recently Kurasov and Sarnak provided a method for construction of one-dimensional counting measures which are FQ (motivated by quantum graphs) using the torus zero sets of multivariate Lee-Yang polynomials. In this talk, I will show that the Kurasov-Sarnak construction generates all FQ counting measures in 1D.

A discrete set on the real line is fully described by the gaps between consecutive points. A discrete periodic set has finitely many gaps. We show that a non-periodic FQ has uncountably many gaps, with a well-defined gap distribution. This distribution is given explicitly in terms of a an ergodic dynamical system induced from irrational flow on the torus.

The talk is aimed at a broad audience, no prior knowledge in the field is assumed.

Based on joint works with Alex Cohen and Cynthia Vinzant.

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