Knowledge Representation and Algorithmic Composition with MultiDiGraphs

Vitor Guerra Rolla

Centre for Informatics and Systems, University of Coimbra (CISUC / UC)
Pólo II - Pinhal de Marrocos
3030-290 Coimbra, Portugal
vitorgr@dei.uc.pt
https://eden.dei.uc.pt/~vitorgr/

Abstract. A relevant aspect of algorithmic composition is the ability to materialize music. Creating musical pieces is a complex endeavour that comprises both, technical knowledge and creativity. This paper proposes an algorithm to compose music based on graph theory. The computer represents the style of a composer within a graph, and re-utilizes such knowledge to compose new music.

Keywords: Algorithm Composition, Graph Theory, Knowledge Representation

1 Introduction

Automatic generation of music can be achieved by enabling the computer to learn the style of a composer and re-utilize such knowledge to compose new music. This work shows a method to construct a *multidigraph* that represent a piece of music. The graph is explored by an algorithm to create new music compositions. A *multidigraph* is a special type of graph that allows multiple directed edges between two nodes [1].

2 Music as a MultiDiGraph

Musical pieces can be seen as dynamical systems with discrete values, i.e., a finite set of musical notes evolving in time [2].

To construct a *multidigraph* for a given piece of music, it is necessary to define nodes and edges. According to [3], every individual note that appears in a piece of music is a node. The edges are defined by connections from one note to another chronologically as the music is played. For example, suppose in a piece of music, the notes start at $t0, t1, t2$,... Suppose note-i and note-j start at time ti and tj, respectively, where $tj > ti$. If no other note starts within (ti, tj), then note-j is said to attune with note-i and a direct edge is defined from note-i to note-j. The duration of this edge is $\delta_t = tj - ti$. Every time when note-j is played right after note-i with a duration δ_t, the weight of this edge is increased

by 1. Each particular edge between two nodes (notes) is labelled with a duration interval δ_t and a weight ψ. Given the aforementioned procedure, it is possible to construct a *multidigraph* and compute the following parameters:

ε $\longrightarrow$ is the set of all edges in the multidigraph.
ε_i $\longrightarrow$ is the set of edges, where i is the source note.
ε_{ij} $\longrightarrow$ is the set of edges, where i is the source note and j is the successor note.
$N\varepsilon_i$ $\longrightarrow$ is the number of edges in ε_i.
$N\varepsilon_{ij}$ $\longrightarrow$ is the number of edges in ε_{ij}.
ψ $\longrightarrow$ is the weight of a particular edge.
Ψ $\longrightarrow$ is the sum of weights from all edges in ε_{ij}.

3 Music composition

A simple interpretation of music composition is a melody which contains a stream of notes. The proposed algorithm starts from a randomly selected node (note) and moves to another node (note) via one of the outgoing edges, i.e., the succeeding node is the ending node of the edge being selected.

The probability (p) of each outgoing edge being selected is proportional: (i) to the number of times that the composer utilized the note-j right after note-i, and (ii) to its weight ψ. Equation 1 describes the calculation of p for each outgoing edge. The first term in the multiplication is the probability of note-j being selected right after note-i, which is given by the number of edges in ε_{ij} divided by the number of edges in ε_i.

$$\forall\ (i,j) \in \varepsilon_i : \\ p = \left(\frac{N\varepsilon_{ij}}{N\varepsilon_i}\right) * \left(\frac{\psi}{\Psi}\right) \tag{1}$$

The second term in the multiplication represents how important is the duration interval δ_t between a pair of notes (i,j). Thus, the probability p is also proportional to the weight of a particular edge (ψ), divided by the sum of weights (Ψ) from all edges in ε_{ij}. The calculation of Ψ is given in Equation 2.

$$\Psi = \sum_{(i,j)\in\varepsilon_{ij}} \psi \tag{2}$$

4 Experiments and Results

The original musical pieces utilized to create *multidigraphs* were the Hungarian Dance N^{o}5 composed by Johannes Brahms and the Minuet composed by Johann Sebastian Bach. It is important to notice that the multidigraph represents only the melody, and do not have rhythmic and motif considerations. Such considerations will be taken into account in a future work. The original musical pieces and the automatic composed musical pieces are available at *http://eden.dei.uc.pt/∼vitorgr/MS.html*.

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