On the coalescence time of reversible random walks

McGill Discrete Maths & Optimization Seminar

Roberto Imbuzeiro Oliveira (IMPA)

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http://arxiv.org/abs/1009.0664 - To appear in Transactions of AMS

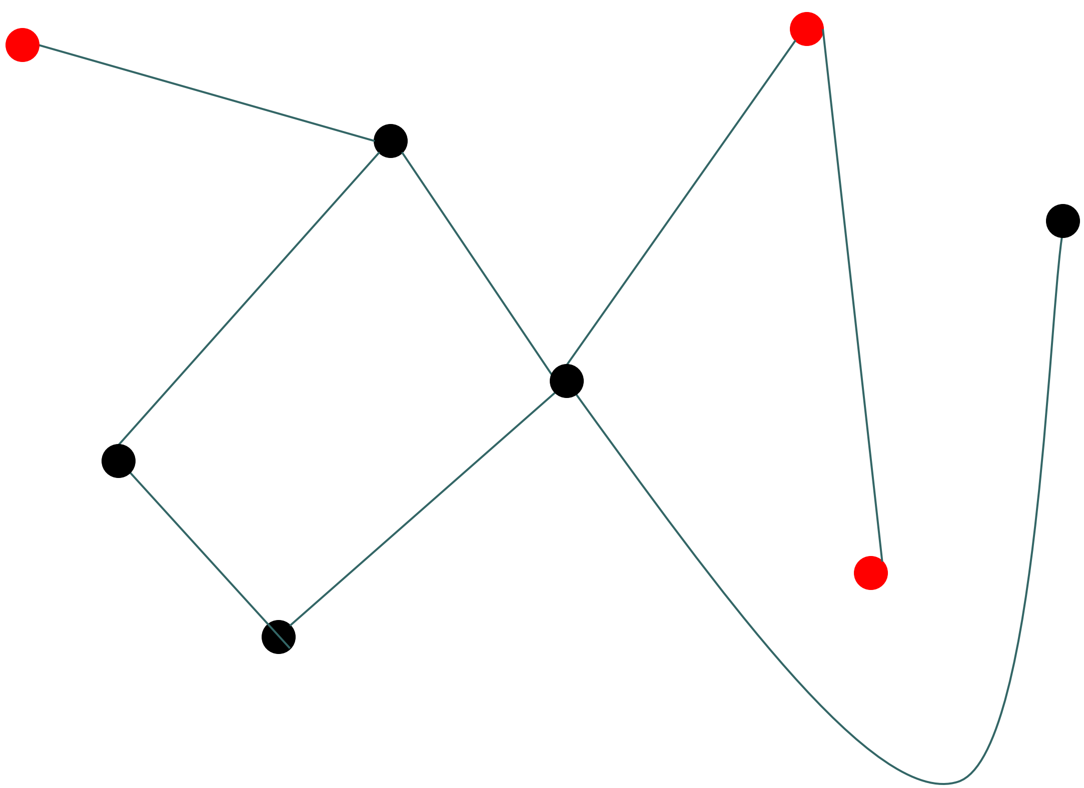
Voter model

Applet link.

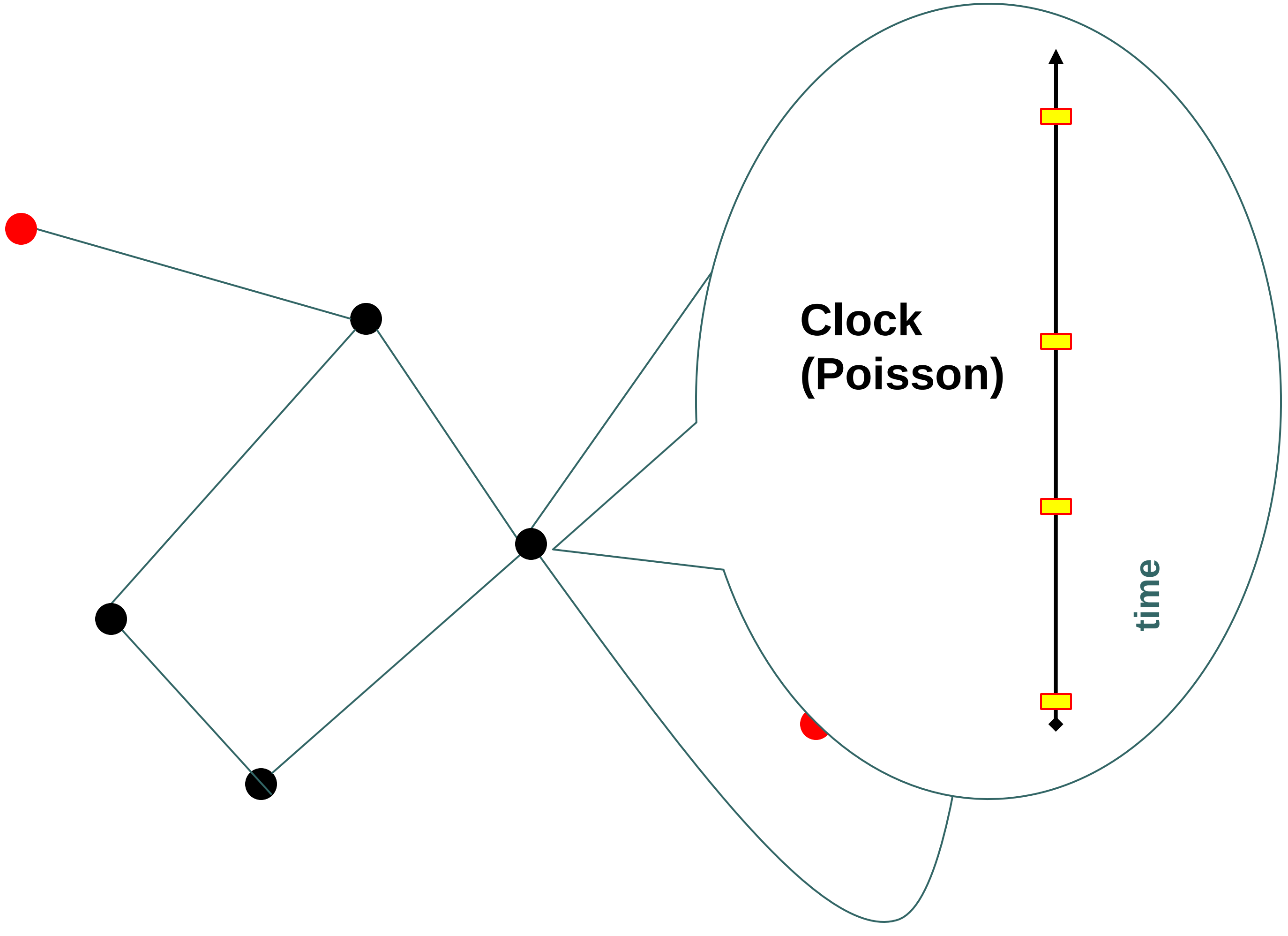
Voter model

- Each vertex v has na alarm clock (Poisson process with rate 1).
- When v's clock rings, it picks a random neighbor and copies its opinion.

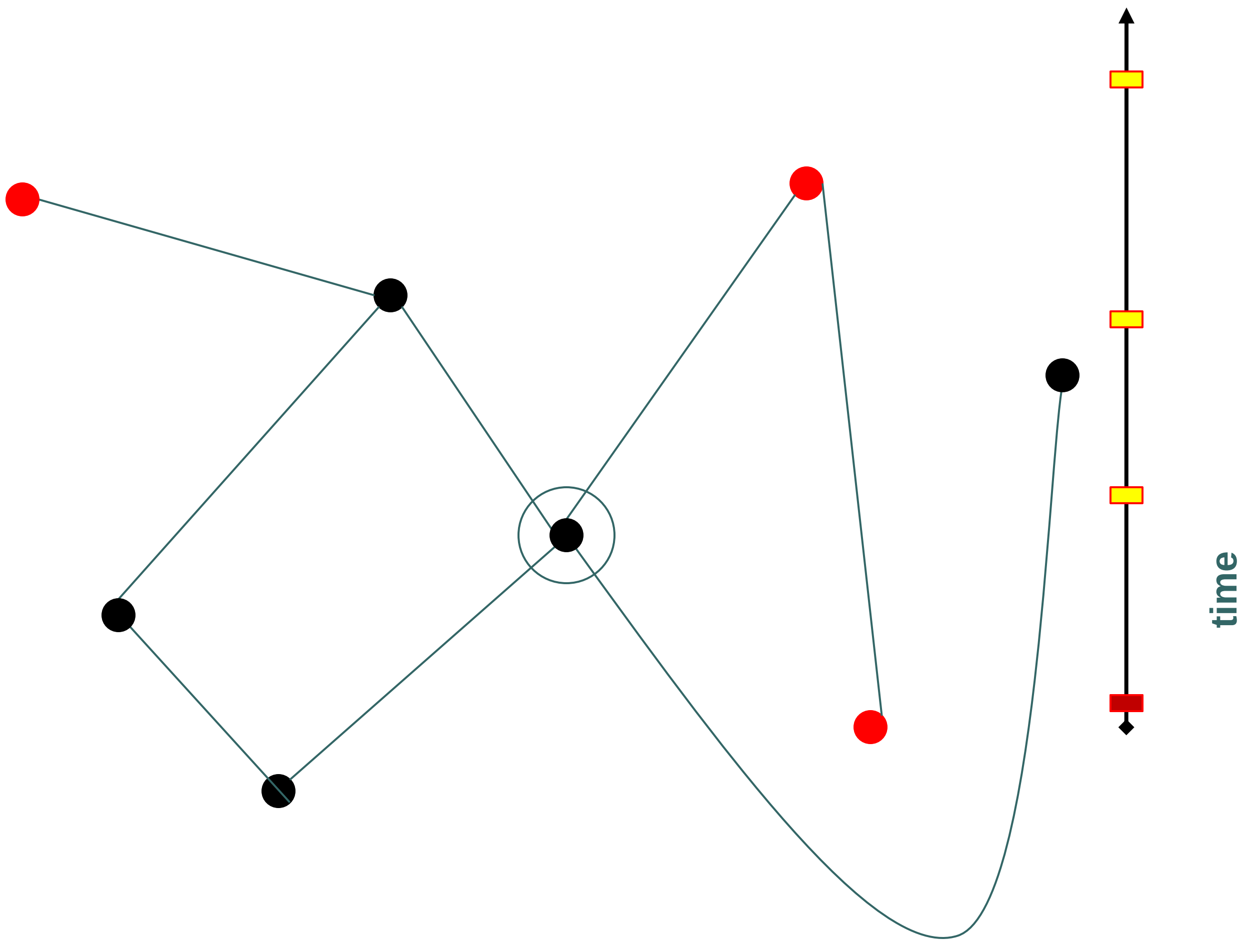
Voter model



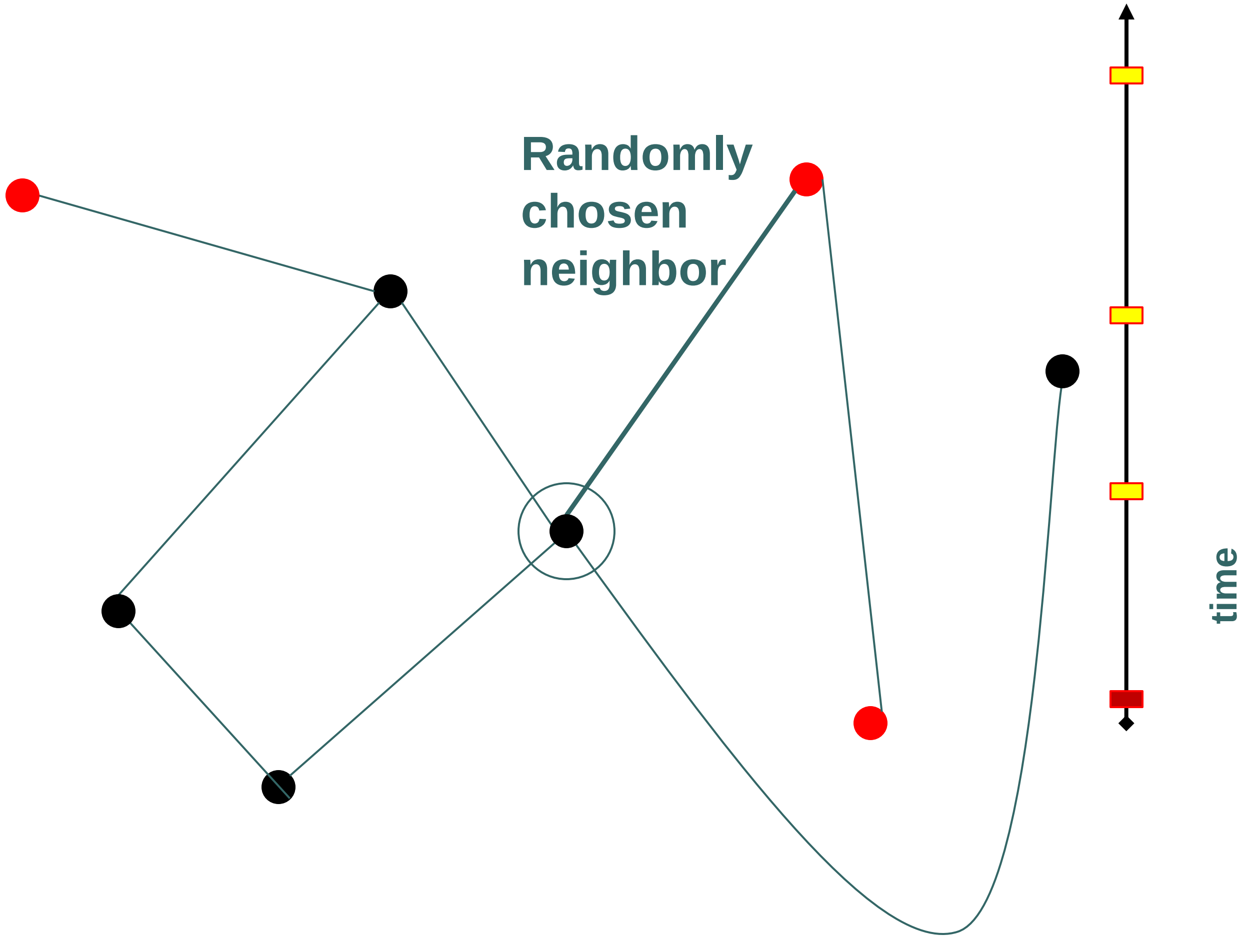
Voter model



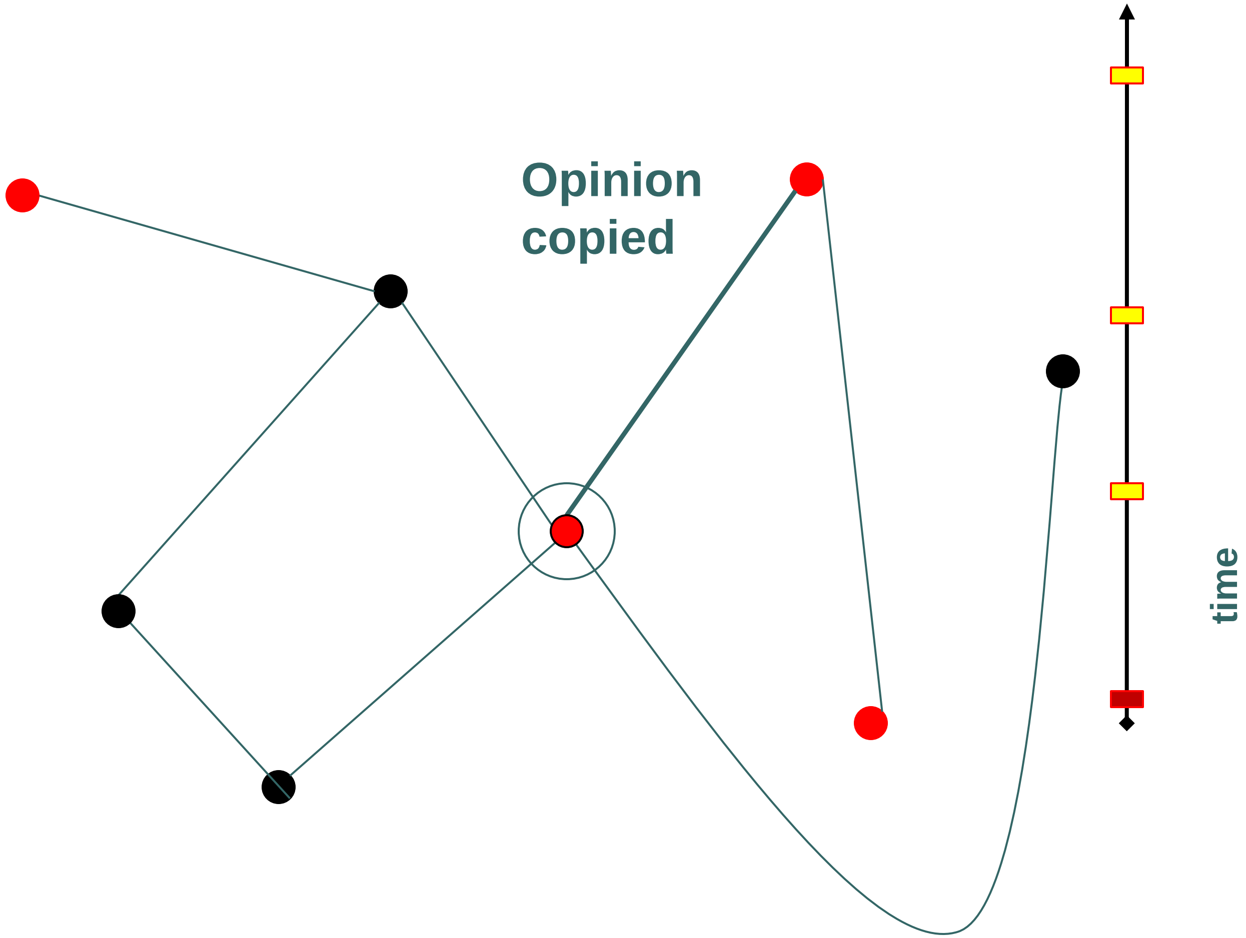
Voter model



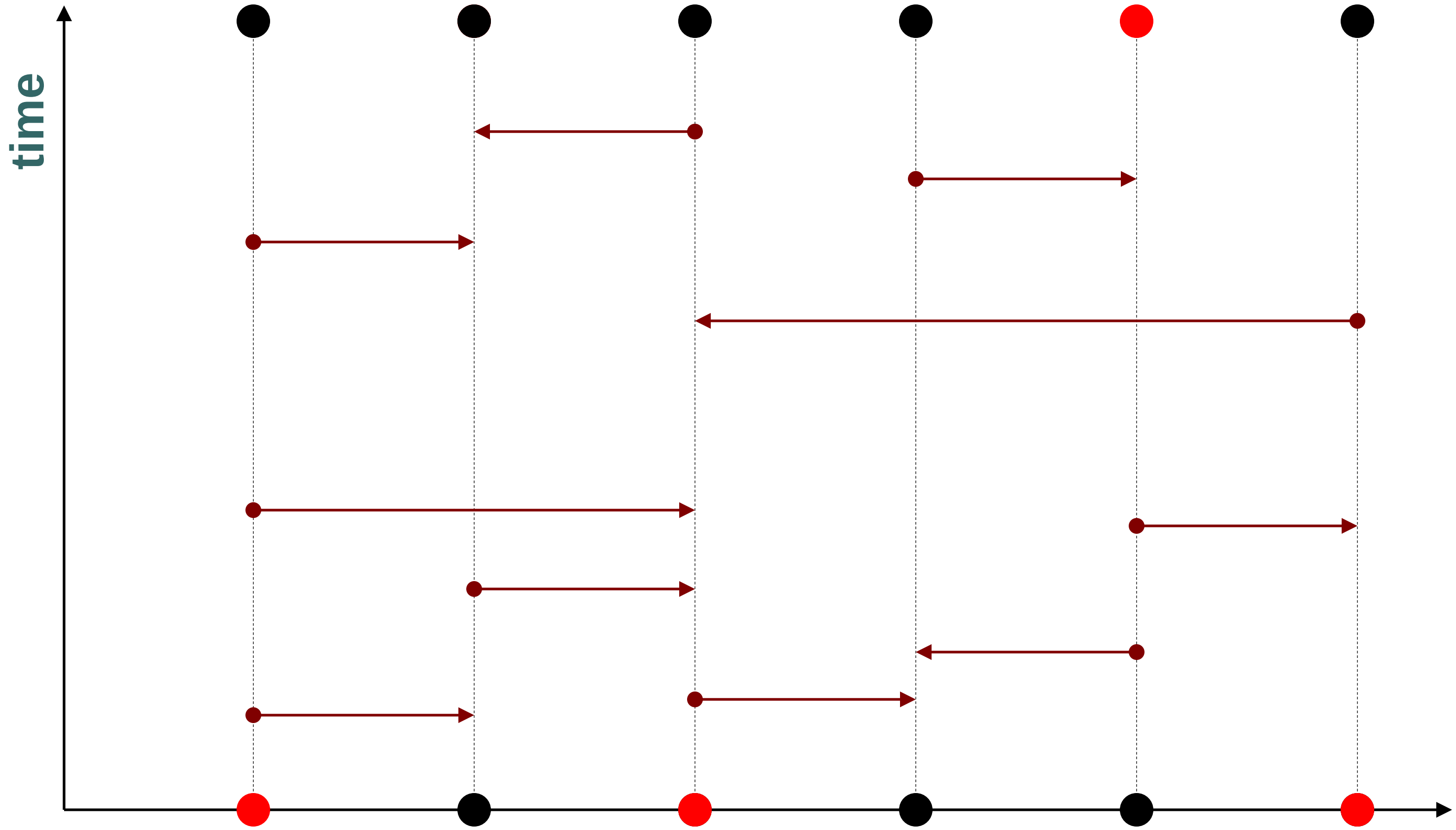
Voter model



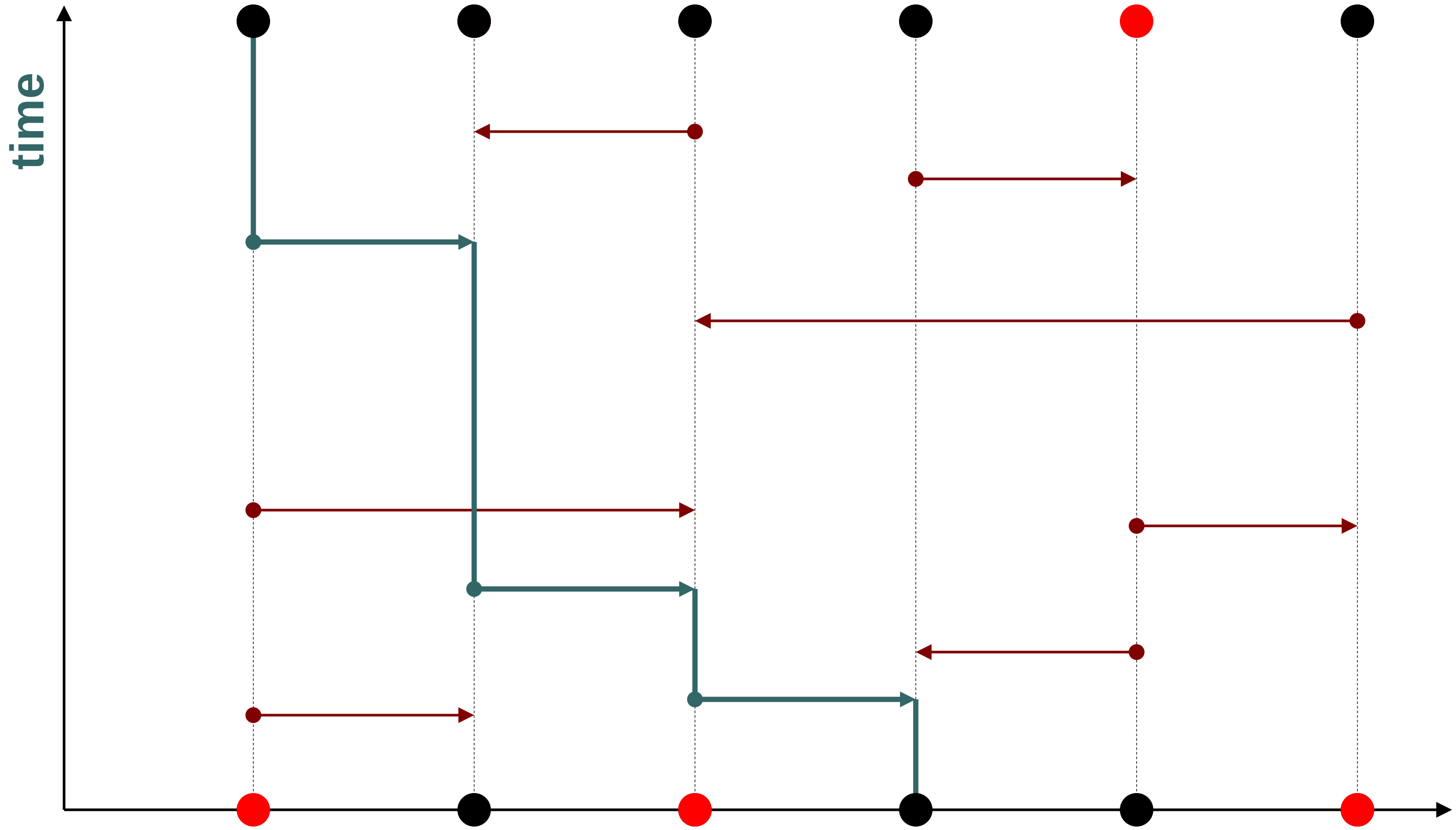
Voter model



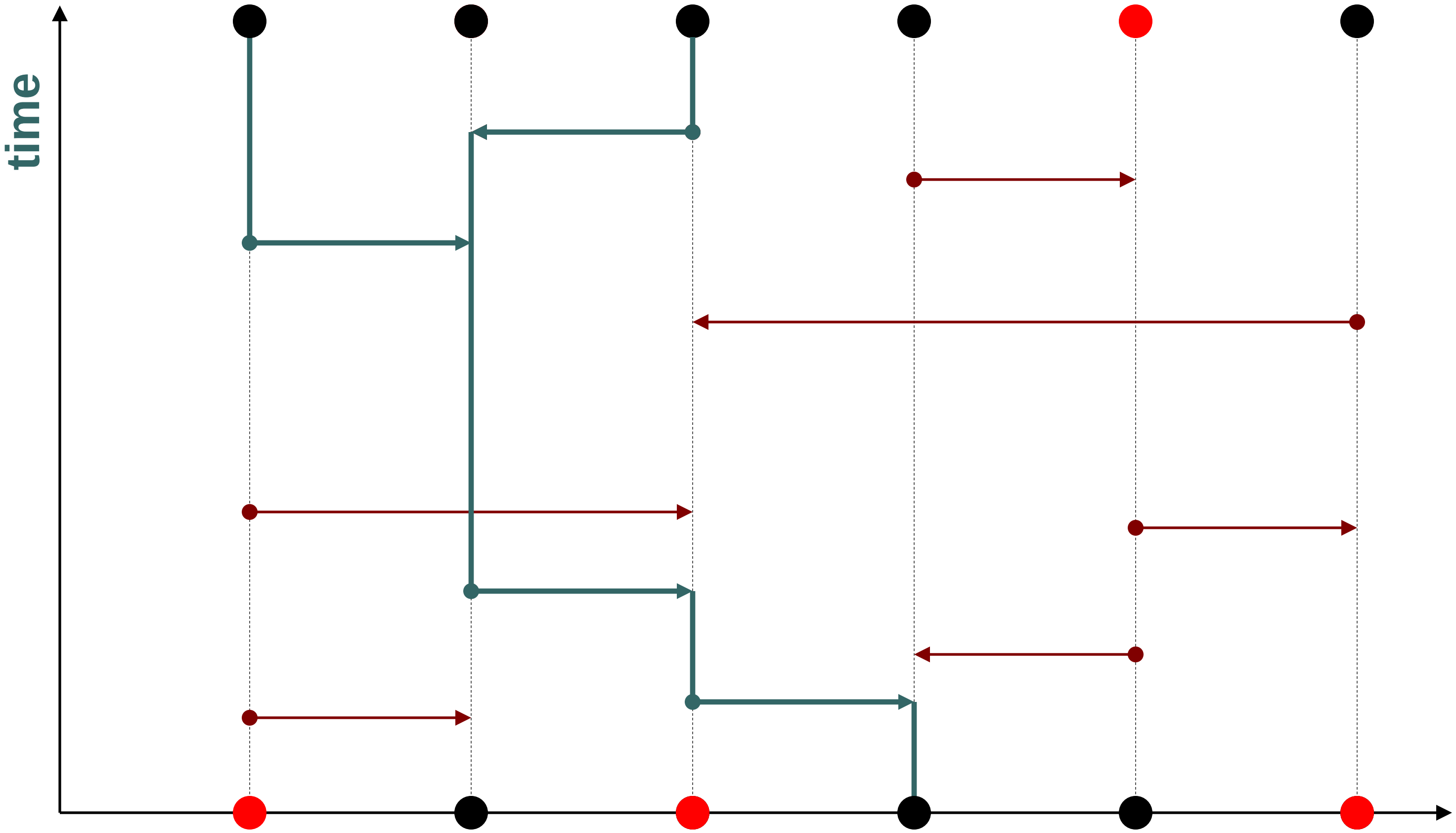
Duality



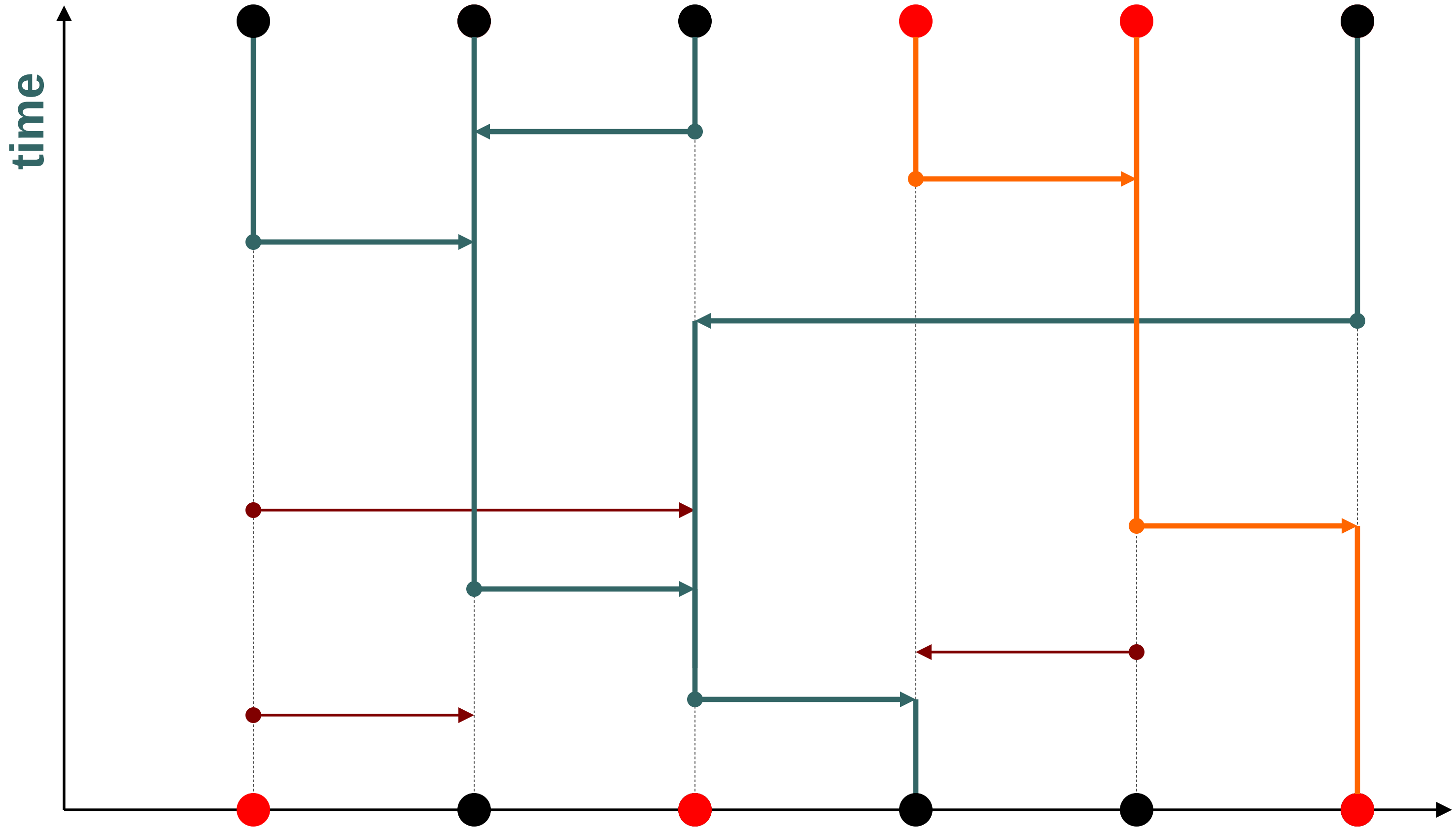
Duality



Duality



Duality



Duality

- Voter model is dual with **coalescing random walks**.

Questions

- How long until full coalescence?
- How long is the time until consensus in the voter model?
- Consensus $=_d$ full coalescence if all opinions are initially different.

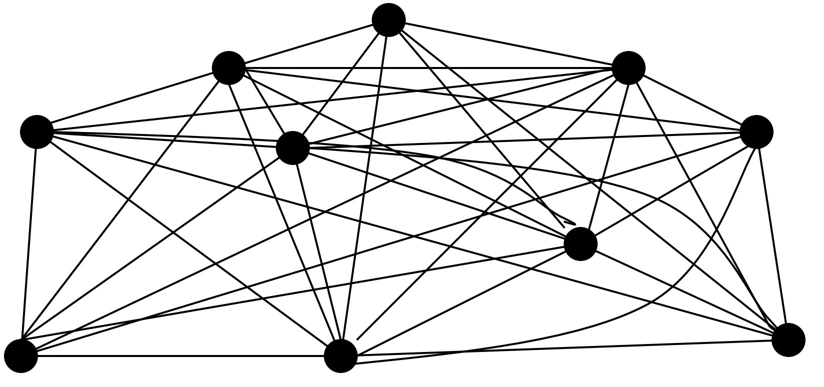
A lower bound

- Expected meeting time of two random walks on G.

Preety good for K_n

- Full coalescence time =

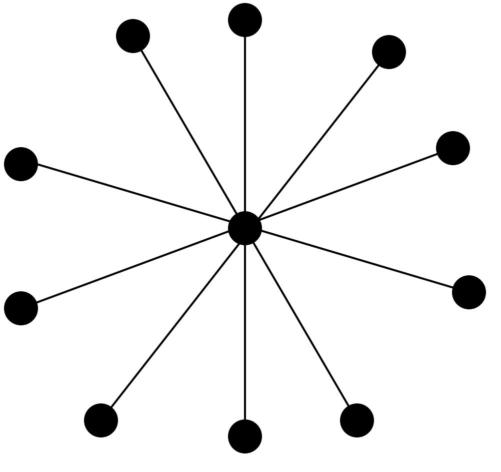
  $= (2-n^{-1}) \times$ (meeting time of 2).



Preety bad for stars

- Full coalescence time =

  = const. (log n) x (meeting time of 2).



Theorem (O'10)

- If G is vertex transitive and connected,

  Full coalescence time ≤

  ≤ const. x (meeting time for 2).

where the constant is universal.

More general result (O´10)

- Let Q be a reversible Markov chain with finite state space. Then:

Full coalescence time ≤

≤ const. x $T_{hit}(Q)$.

where $T_{hit}(Q)$ is the largest expected hitting time for a state of the chain.

About the result

- Aldous'91 proved:

  Meeting time of 2 ≤

  ≤ const. x $T_{hit}(Q)$,

  Right order of magnitude when Q is transitive, and in many other cases.

- Aldous&Fill'94 asked whether the same bound was true for full coalescence.