Warping and Morphing Techniques

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The Transformation Process

User

Specification

Representation

Computation

Results
Objective

- Some specific warping and attribute combination techniques:
  - computation/specification dependencies
  - applicability to different types of GO’s
  - implementation guidelines
Warping Specification

- Coordinate system mapping definition
- Simply maps areas in the domain of definition
- Graphical Object independent
Warping Computation

- How can the map be extended to the entire domain?
- How can the map be applied to a GO?
- Ideally the solutions are not coupled
Extending the Specification

- Graphical object $f : U \subseteq R^n \rightarrow R^m$
- Mapping for all points of the domain: $W : U \rightarrow U$
- Interpolation/extrapolation of sparse data
Map Application

- Apply given $W$ to GO

- Depends on GO:
  - Polyhedral objects
  - Images and volumes
  - Implicit objects
  - etc.
Mapping Polyhedral Objects

- Replace each point \( p \) by \( W(p) \)
Mapping Implicit Objects

- Given implicit object with characteristic function

\[ f : \mathbb{R}^n \rightarrow \mathbb{R} \]

- Warped implicit object is

\[ g(x) = f(W(x)) \]
Mapping Implicit Objects

\[ f(x, y, z) = x^6 + y^6 + z^6 - 1 \]

\[ g(p) = f(W(p)) \]
Mapping Images (or Volumes)

- Grid requires resampling
  - Sampling and reconstruction problems
- Forward or inverse mapping
Warping Techniques

- Barycentric mapping
- Field-based mapping
- Radial basis functions - RBF
- Free-form deformation - FFD
- Multi-pass spline mesh
- Physically-based warping
Barycentric Mapping

- Triangulate the specification
  - Partitions the domain
- Triangle pair mapped using barycentric coordinates
Barycentric Coordinates

\[ \lambda_3 = 0 \]
\[ \lambda_2 = 0 \]
\[ \lambda_1 = 0 \]

\[ p = \lambda_1 a + \lambda_2 b + \lambda_3 c \]
\[ p' = \lambda_1 a' + \lambda_2 b' + \lambda_3 c' \]

\[ \lambda_1 + \lambda_2 + \lambda_3 = 1 \]
Barycentric Mapping

- Triangle mesh foldover
  - Invalid partitions
- Discontinuous along edges
  - Each triangle mapped independently
- Higher order triangle interpolations
Field-based Mapping

- Each feature pair defines one mapping

- 2D or 3D features: points, lines, boxes
Field-based Mapping

- Final map is a weighted average

\[ W(p) = p + \frac{\sum w_i \Delta p_i}{\sum w_i} \]

\[ w_i = \left( \frac{l_i^p}{a + d_i} \right)^b \]

\[ \Delta p_i = W_i(p) - p \]
Field-based Mapping

\[ p \]

\[ F_1 F_2 \]

\[ F_1' F_2' \]

Mapped point
Field-based Mapping Example
Field-based Mapping

- Global mapping
  - Modify weight function
- Ghosting
- Singularities at crossovers
Radial Basis Functions

- **Affine + Radial:**
  \[ W(p) = A(p) + R(p) \]
  \[ A(p) = Mp + b \]
  \[ R(p) = (R_x(p), R_y(p)) \]

- **Radial basis function:**
  - Function of distance to anchors
  \[ g : R \rightarrow R \]
Radial Basis Function

■ Radial component:

\[ R_i(p) = \sum_{i=1}^{N} a_i g\left(\| p - p_i \| \right) \]

■ Given specification with \( N \) anchors \( p_i, p_i' \)

\[ W(p_i) = p_i' \]

◆ Solve linear system to obtain coefficients \( a_i \)
Radial Basis Function Example

\[ g(t) = e^{-t^2 / \sigma^2} \]
Radial Basis Functions

- Locality can be controlled
- Elastic component
  - Anchor points displacement limited by locality factor
Free-form Deformation - FFD
Free-form Deformation - FFD

- **Bernstein polynomial:**
  \[ B_i^d(v) = \binom{d}{i} v^i (1 - v)^{d-i} \]

- **Warping function:**
  \[ W(s, t, u) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} P_{i,j,k} B_i^{n-1}(s) B_j^{n-1}(t) B_k^{n-1}(u) \]
Free-form Deformation

- Global
- High degree polynomials
- Control points in a grid
Multi-pass Spline Mesh Warping

- Images or volumes
- Separable transformation
  - reduces problem to 2 (or 3) 1D problems
Horizontal Pass Overview

- Decompose displacements
- Construct vertical splines without vertical displacements
- Intersect scanline with vertical splines
- Construct spline for scanline mapping
Decompose & Construct Splines

Horizontal displacements

Vertical Splines
Scanline Intersection

- Intersect splines with scanline
Scanline Map

- Construct spline with intersections
  - 1D map
2-pass Spline Mesh Example
Multi-pass Spline Mesh

- Efficient
- Bottleneck problem
- Laborious specification
- Splines cannot cross
Physically-based Warping

- Deformation derived from physical models:
  - Springs
  - Fluids
  - Cloth
  - etc.
Introduction

Warping Techniques

Blending Techniques

Attribute Combination Techniques

Conclusion
Blending Techniques

- Morphing
- Compatibilization of shape of GO’s
  - different genus
  - different combinatorial topology
Blending Techniques

- Physically-based blending
- Merging of Combinatorial Structures
- Exponential blending
Physically-based Blending

- 2D or 3D polyhedral objects
- Energy Model:
  - Wireframe object
  - Optimization problem
  - Minimize work for stretching and bending
  - Vertex insertion for object compatibilization
Physically-based Example

- Automatic feature association
Merge of Combinatorial Structures

- Two 2D or 3D polyhedral objects
  - Different combinatorial structures

- Overview:
  - Projection of structures onto sphere
  - Combination of structures (clipping)
  - Reprojection onto original geometry
Merge Example
Exponential Blending

- Two implicitly defined objects
  - different topologies

- Exponential blending:

\[ B_t(x, y) = -\log[(1-t)e^{-x} + te^{-y}] \]
Exponential Blending Example
Introduction
Warping Techniques
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Attribute Combination

- Attributes
  - Properties associated with object shape
    - Color, texture, depth, vector fields, etc.
  - GO’s shapes must be registered
  - GO’s topologies must be compatible
Attribute Combination

- Dependent on attribute’s nature:
  - Linear interpolation
  - Slerping
  - Exponential blending
  - etc.

- There may be interdependencies
Example: Texture Combination

Color Space

Parameter Space
Applications

- Morphing is not just a special effect:
  - Lens distortion correction
  - Graphical objects modeling
  - Motion capture data interpolation
  - Accelerated Rendering
  - etc.
Example: Image-based Rendering

- Images are pre-rendered
  - Triangulated cubical environment map
  - Attributes: color, depth, fragment quality

- Real-time morphing:
  - Warping by visualization parameters
  - Complex attribute combination
Cubical Environment Map
Blending two nodes
Conclusion

- **Matching specification & computation**
- **Warping techniques**
  - Reconstruction of the transformation
- **Blending techniques**
  - Topology merge
- **Attribute combination techniques**