Hierarchical 4–k Meshes

Luiz Velho
Jonas Gomes

Visgraf Laboratory
IMPA – Instituto de Matemática Pura e Aplicada
Rio de Janeiro, Brasil

Outline

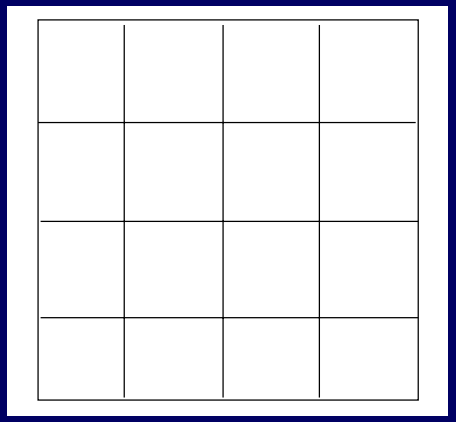
- Background on 4-8 Meshes
- Hierarchical 4-k Meshes
- Representation and Operations
- Construction Methods
- Recent Developments

Background

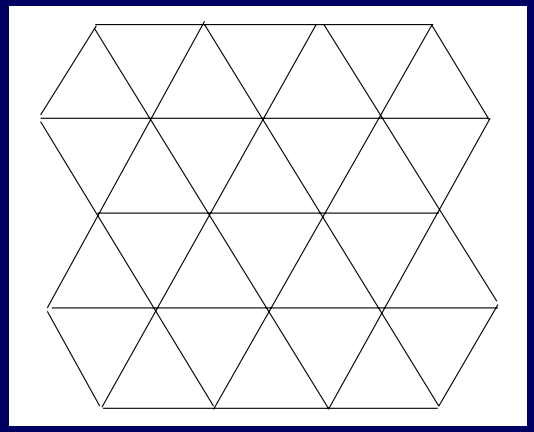
- Origins of Classical 4–8 Meshes
  - Laves Tilings
  - 4–direction Meshes
  - Quincunx Lattices
- Generalizations of 4–8 Meshes
  - Semi-Regular 4–8 Meshes
  - Quasi-Regular 4–8 Meshes
  - Hierarchical 4–k Meshes

Uniform Partitions of $\mathbb{R}^2$

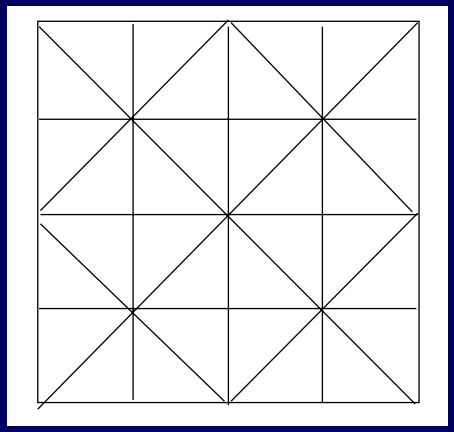
- Laves Tilings



$[4^4]$



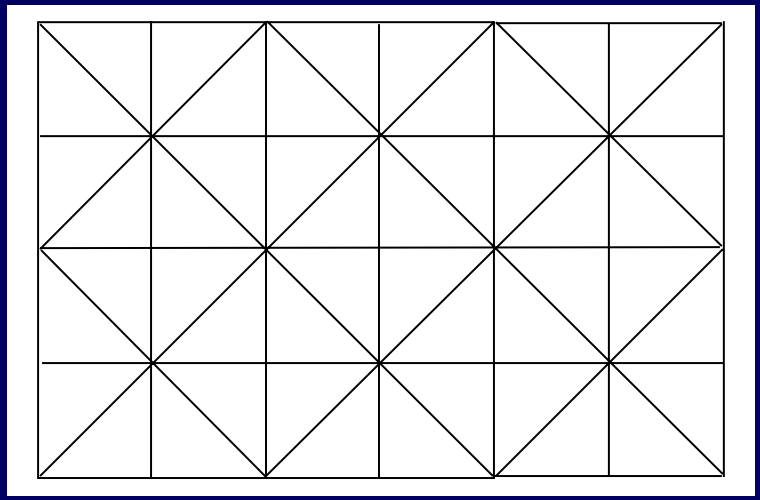
$[6^3]$



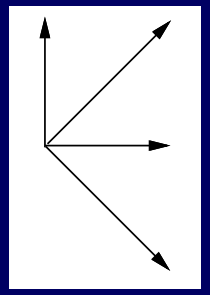
$[4.8^2]$

* Symmetry: Crystallographic Groups

Four Direction Meshes



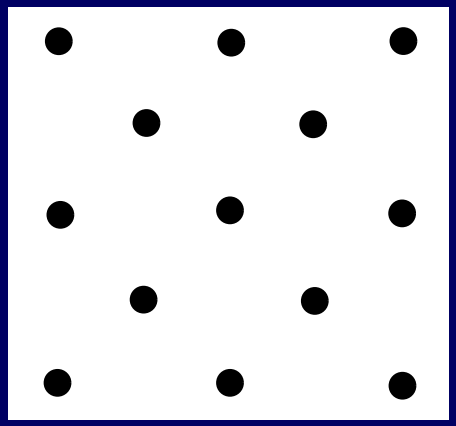
- Generated by the vectors $(e_1, e_2, e_1 + e_2, e_1 - e_2)$

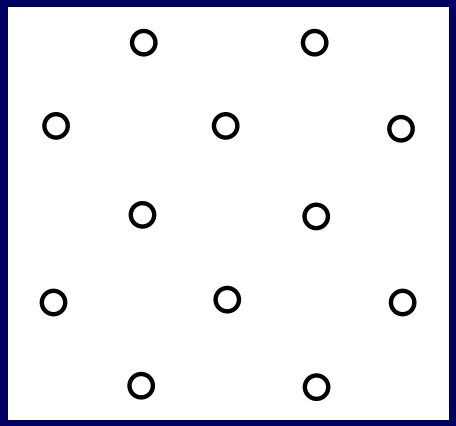


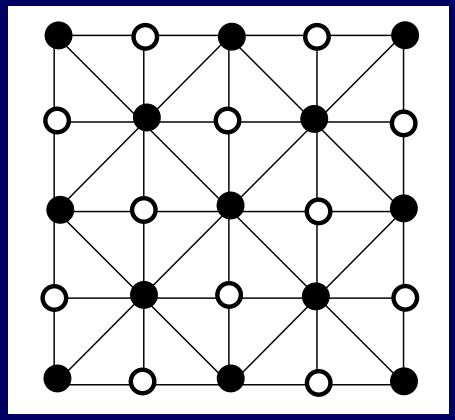
Quincunx Lattices

- $Q = \{Mx; x \in \mathbb{Z} \times \mathbb{Z}\}$, where M is the *quincunx matrix* $M = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

- Two Interleaved Quincunx Lattices

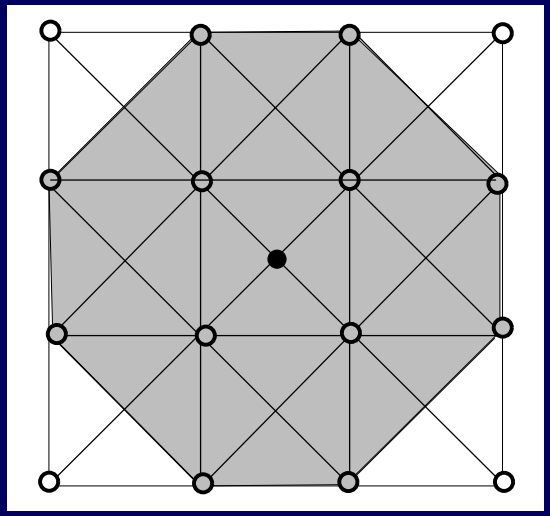






Four Direction Box Splines

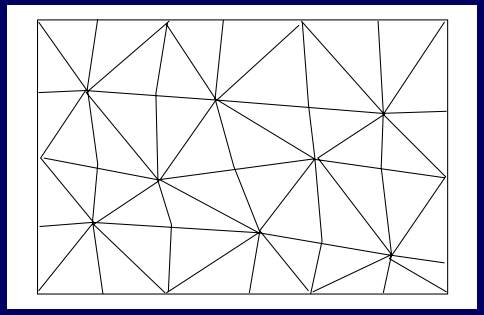
- C^1 basis function generated by the convolution set $\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$



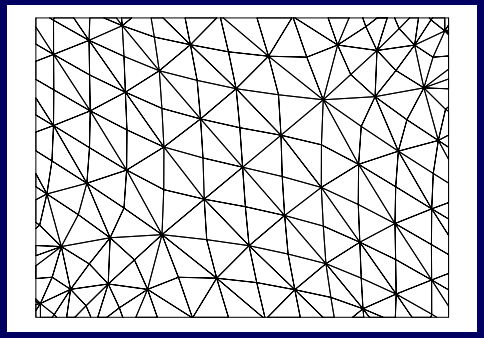
* *Defined on Quadrilaterals*

Generalization of 4–8 Meshes

- Regular 4–8 Meshes
  - 4–8 Topology
    ($\deg(v) = 4, 8$)



- Semi-Regular (Quasi-Regular) 4–8 Meshes
  - Isolated Extraordinary Vertices
    ($\deg(v) \neq 4, 8$)



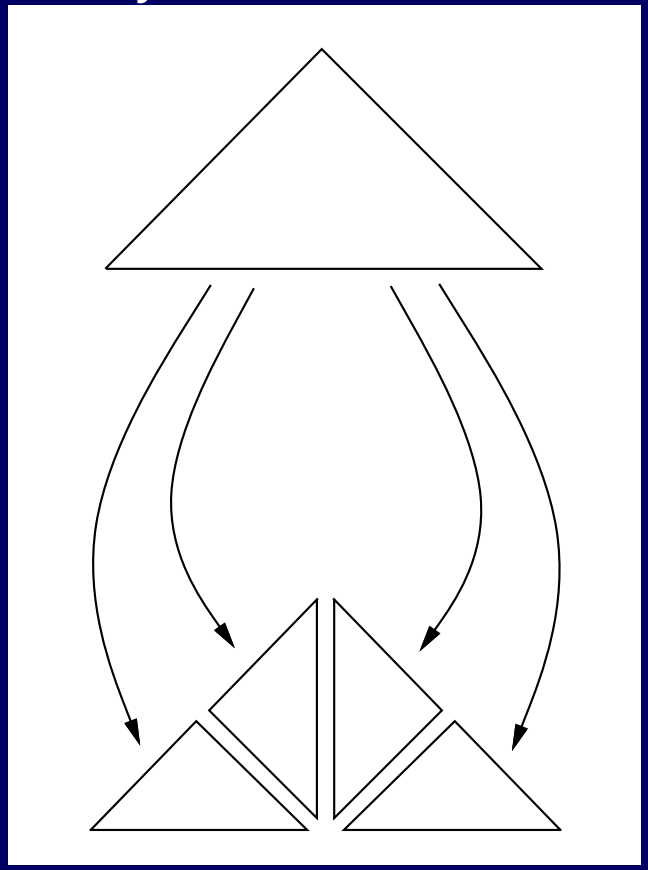
* *Most of the properties of 4–8 Meshes*

Hierarchical 4–k Meshes

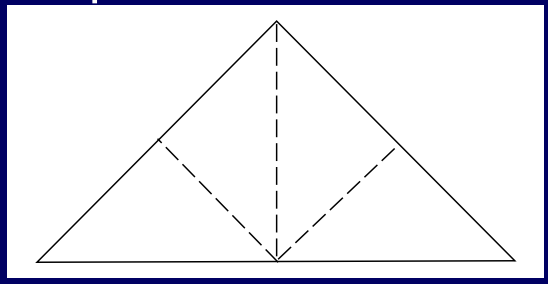
- Construction Methods
  - Direct Refinement
  - Red-Black Refinement
- Representation
  - Multiresolution Representation
  - Variable Resolution Representation

Direct Refinement

- Quaternary Subdivision

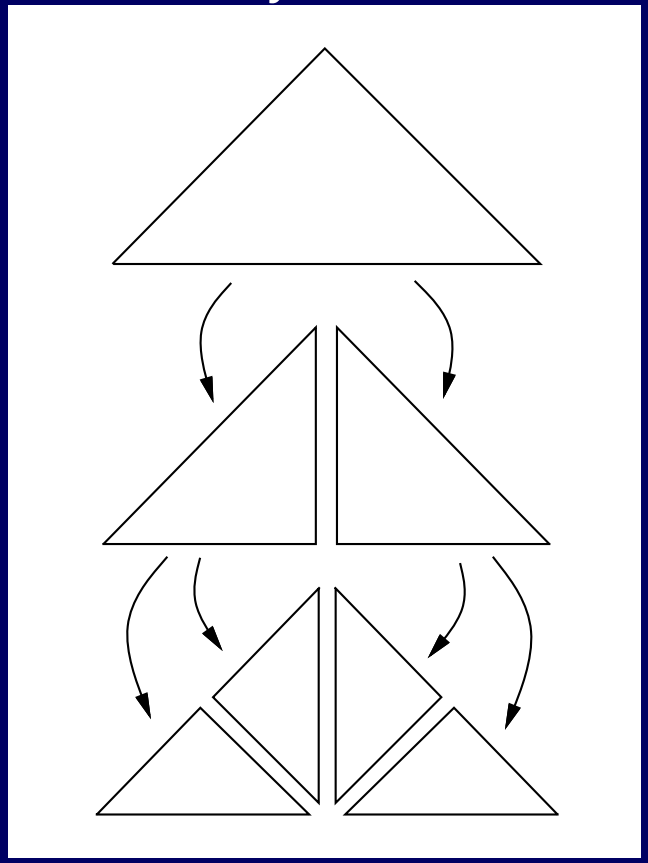


- Template

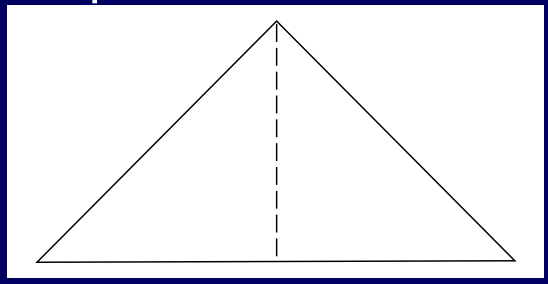


Red-Black Refinement

- Recursive Binary Subdivision

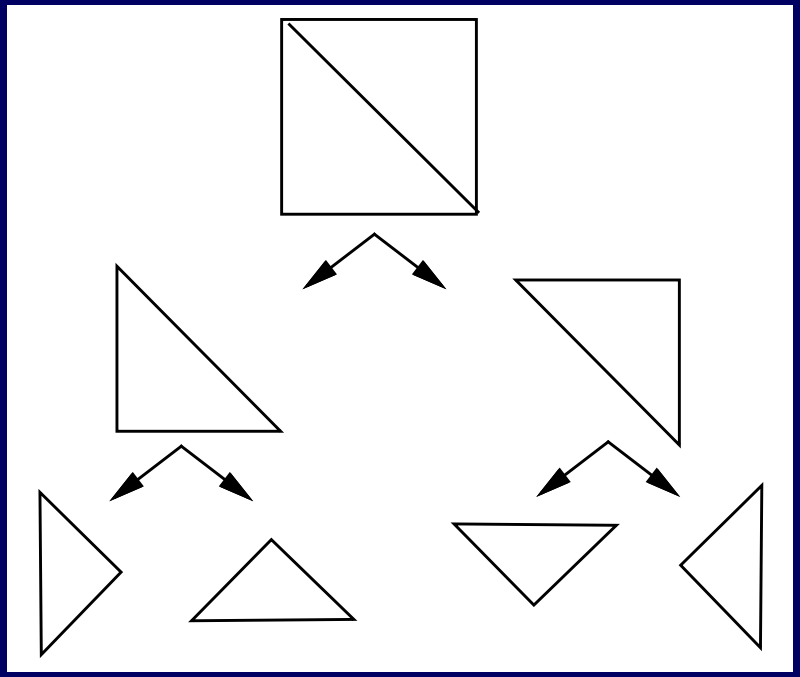


- Template



Hierarchy of Right Triangles

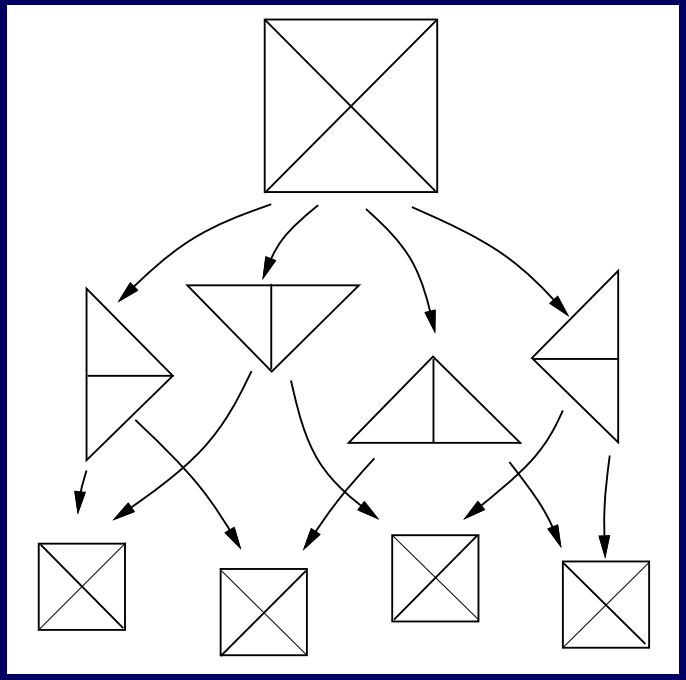
- Binary Tree Representation



* *Regular Case*
(Terrain)

Variable Resolution Multi-Triangulation

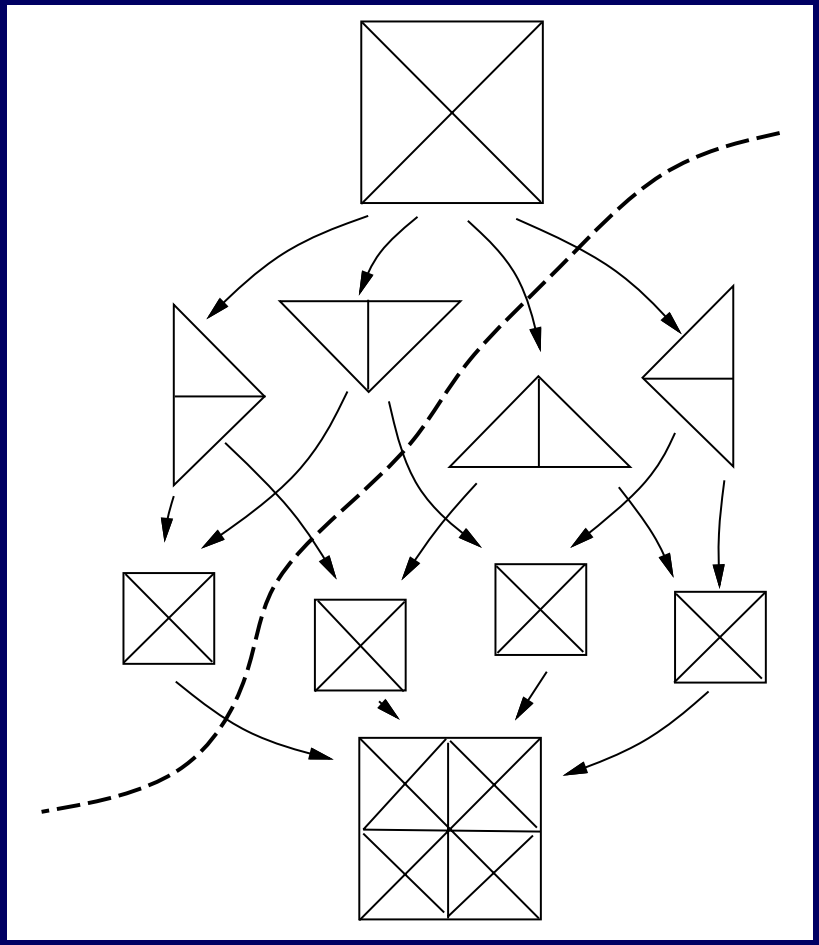
- Dependency Graph (DAG)



* *Semi-Regular Case*

Query Operations

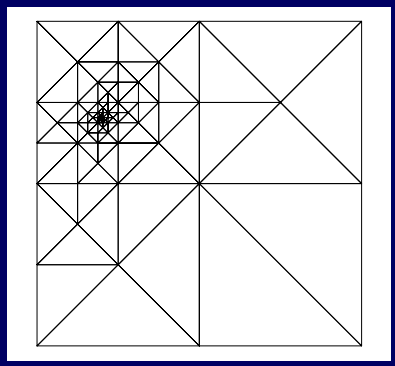
- Cut in the DAG

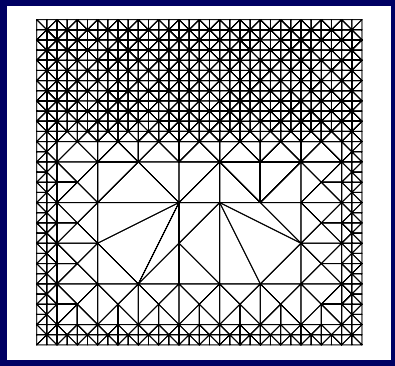


- Definition:
  - Threshold Function
  - Focus Set

Mesh Extraction

- Adapted Meshes
  - Geometry Approximation
  - View-Dependent
  - Etc.

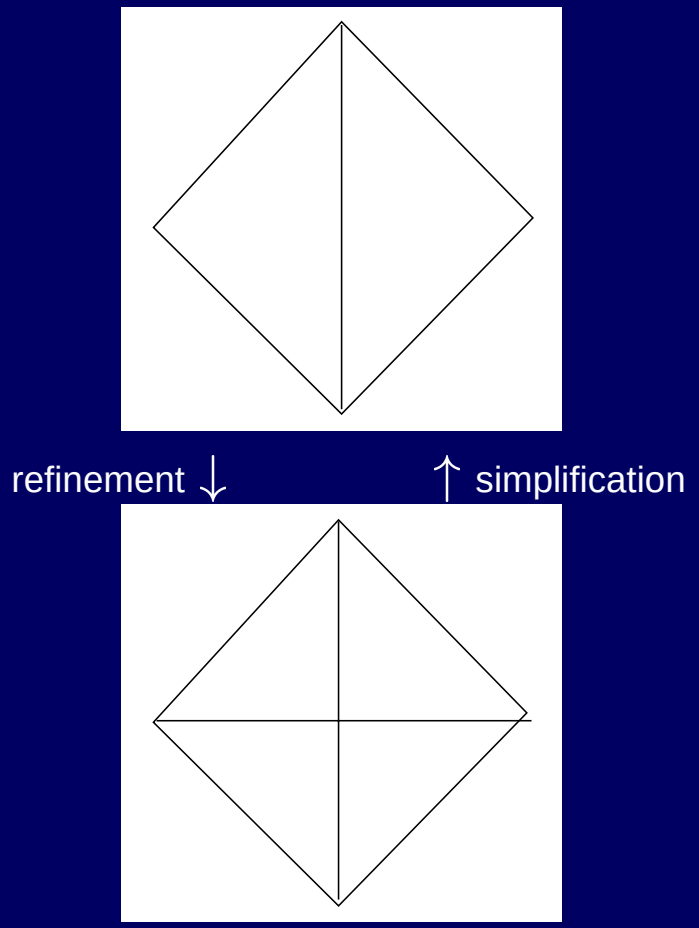




* *Consistency by Construction*

Construction Methods

- Local Modifications
  - Refinement
    (Edge Bisection)
  - Simplification
    (Half Edge Collapse)

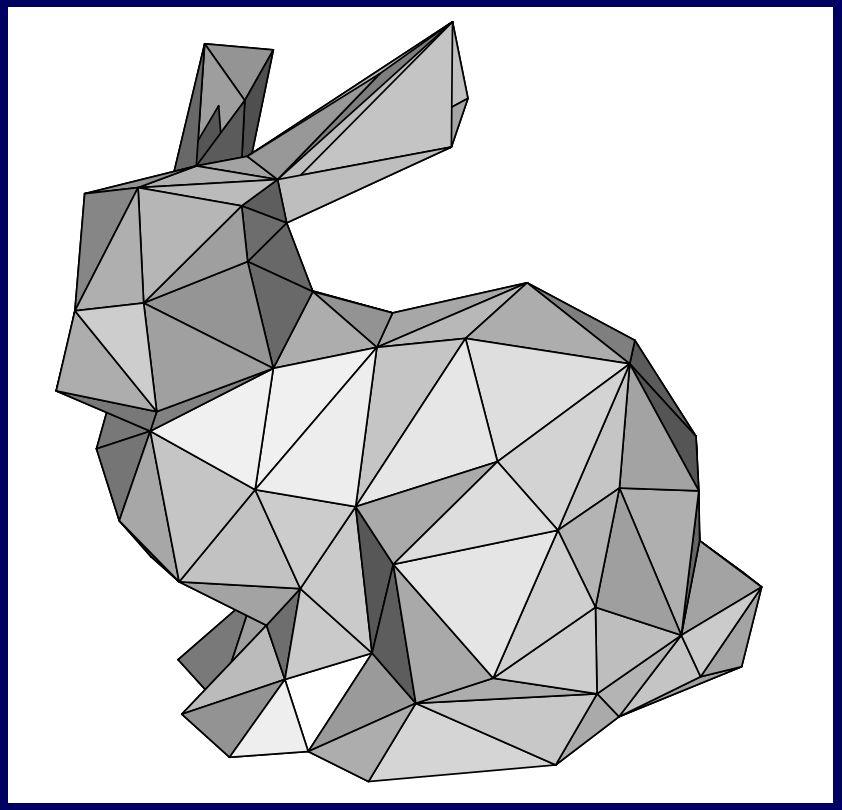


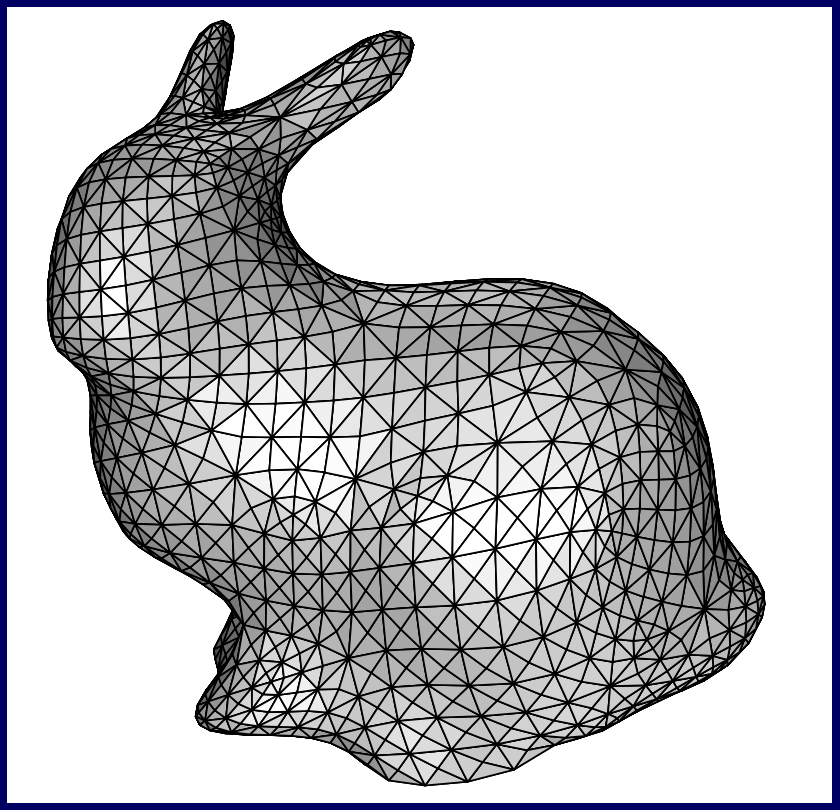
New Developments

- Refinement Methods
  - Topology-Based (Semi-Regular)
    Generalized 4 direction Box Splines
  - Geometry-Sensitive (Quasi-Regular)
    Quasi 4–8 Subdivision Surfaces
  - Adaptive (Irregular)
    Tesselation of Parametric and Implicit Surfaces
- Simplification Methods
  - Topology Preserving (Irregular)
    2-Manifold Triangle Meshes

Four Direction Box Splines

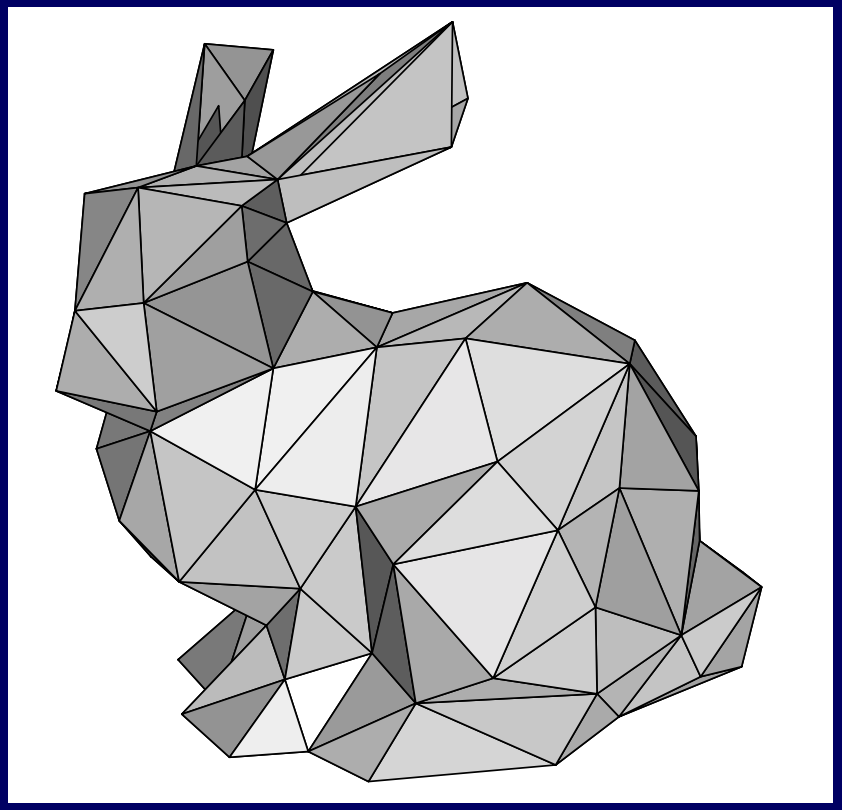
- Interleaved Edge Subdivision

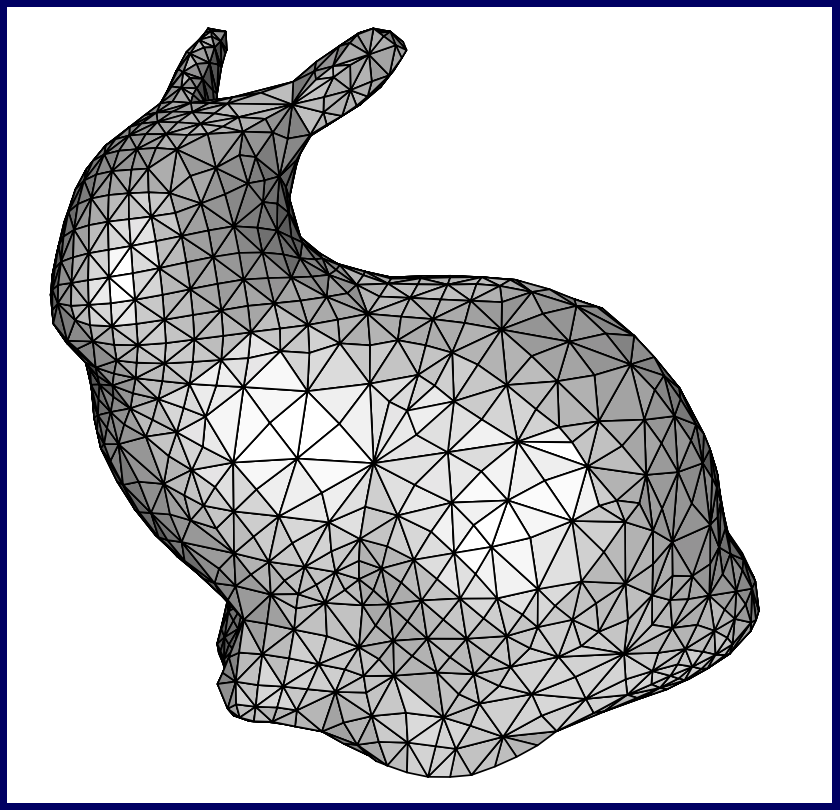




Quasi 4–8 Subdivision Surfaces

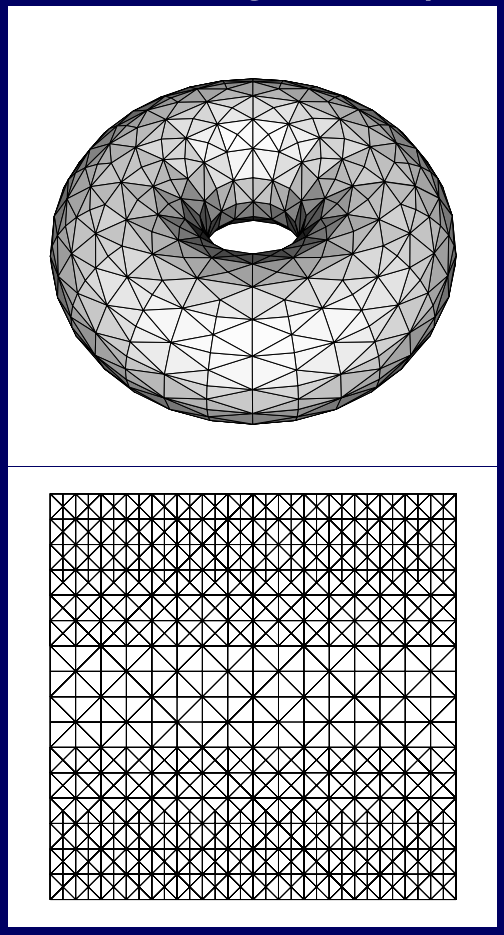
- Longest Edge Bisection



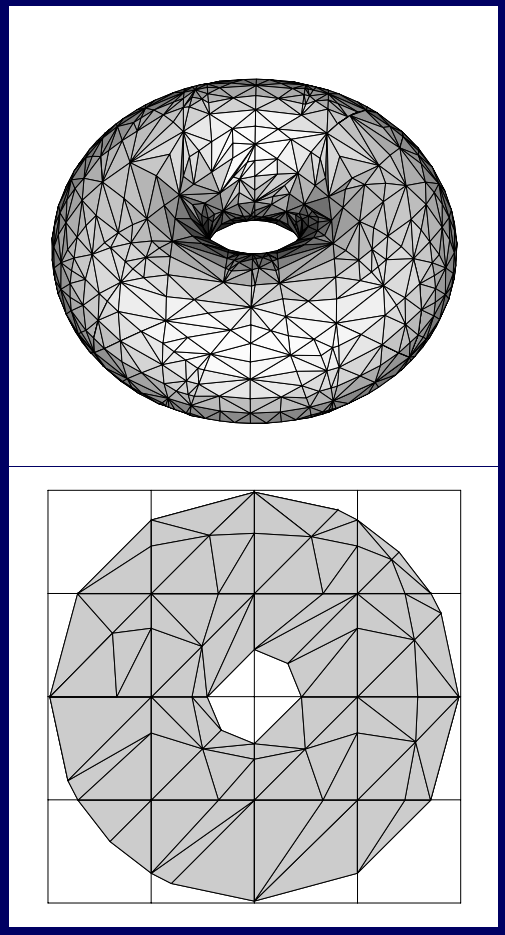


Adaptive Surface Tesselation

- Multiresolution Edge Sampling



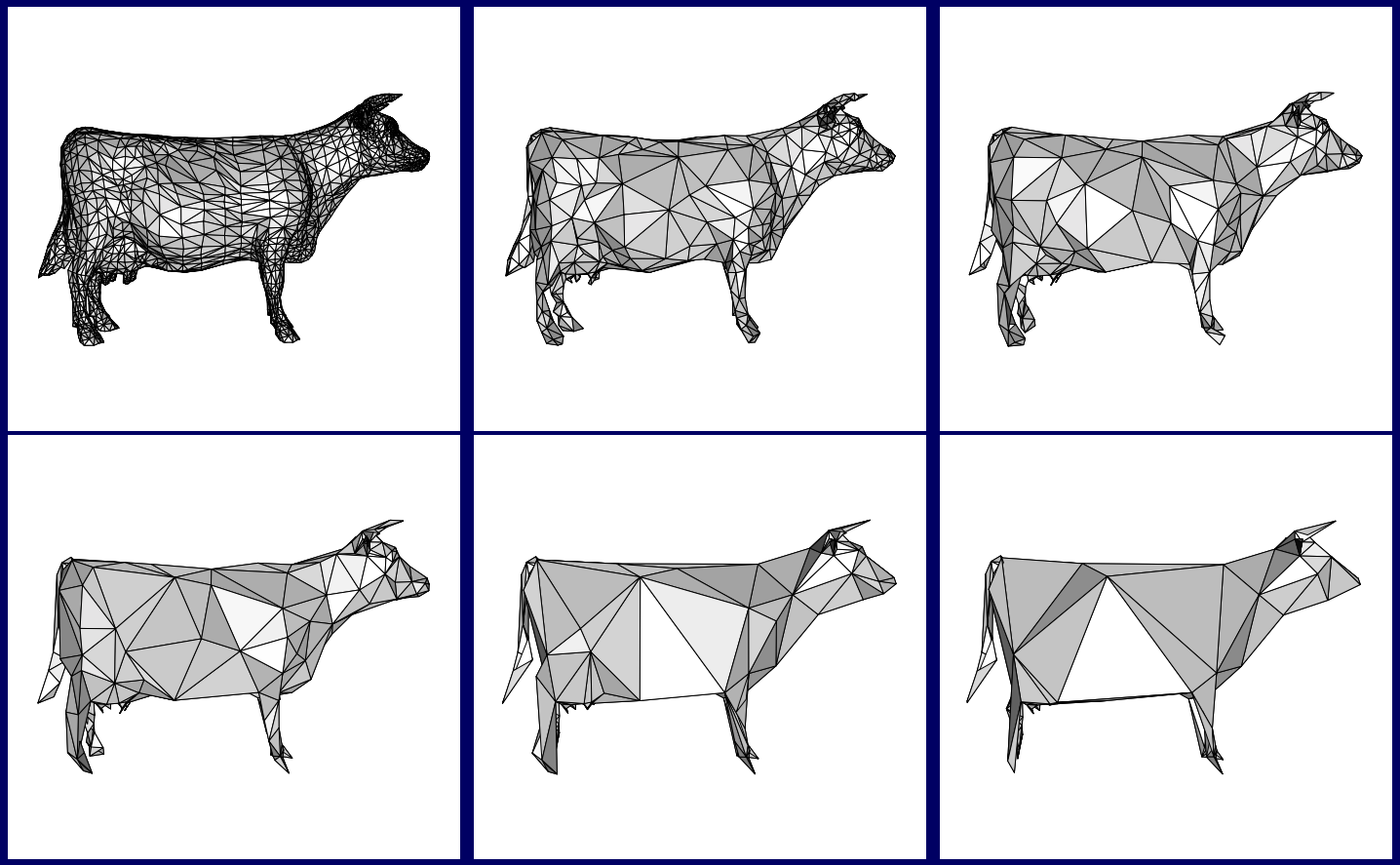
parametric surface



implicit surface

Surface Simplification

- Half-Edge Collapse and Edge Swap



Future Work

- Hierarchical Parametrizations
- Multiresolution Decomposition
- Space-Filling Hamiltonian Paths
- Integrated Framework