Quadrilateral Meshing using 4-8 Clustering

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Outline

- Problem Statement
- Motivation
- Description of the Method
- 2D / 3D Results

Problem Statement

Generate a Quadrangulation a 2D Domain

- Two Cases:

  1. Boundary $\rightarrow$ Quadrangulate Interior

  2. Triangulation $\rightarrow$ Convert to Quadrangulation

* We will consider case 2

Motivation

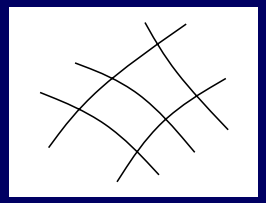
Quadrilateral Meshes are Required in many Applications

- Engineering
  - FEM Analysis
- CAD / CAM
  - NURBS
- Graphics
  - Subdivision Surfaces (4-8 Box Splines)

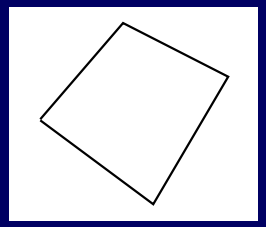
Intuition

Difficult Problem

- Find Main Direction Curves



- Build Well-Shaped Elements



- More Global than Triangulation (Edelsbrunner)

Basic Approaches

- **Global Solution**
  (Optimization + Heuristics)
  - Pros: Good Meshes, Few Elements
  - Cons: Costly, Complex
- **Local Solution**
  (Re-tesselation + Rules)
  - Pros: Fast, Simple
  - Cons: Bad Meshes, Many Elements

Proposed Solution

Hybrid Approach

- Global
  - Find Large Areas with Good Quad-Patch Structure
  - (4-8 Two-Face Clustering)
- Local
  - Join Areas with Compatible Structure
  - (Catmull-Clark Subdivision)

Algorithm

(1) Find an independent set of two-triangle clusters, and identify the remaining isolated triangle faces;

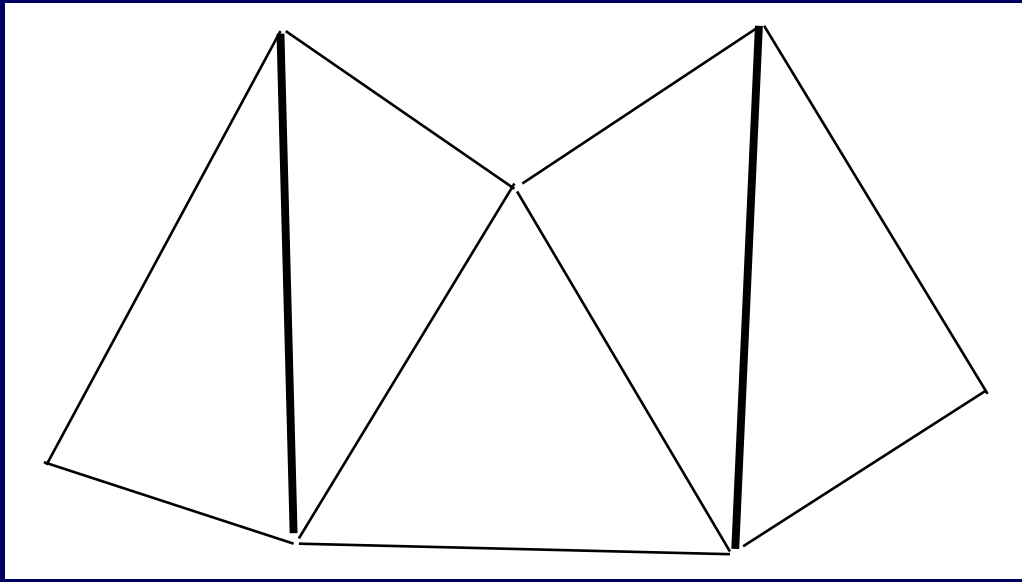
(2) Perform a hybrid binary subdivision step;

(3) Perform one step of binary subdivision;

(4) Remove internal edges of triangulated blocks.

Step 1 - Cluster Marking

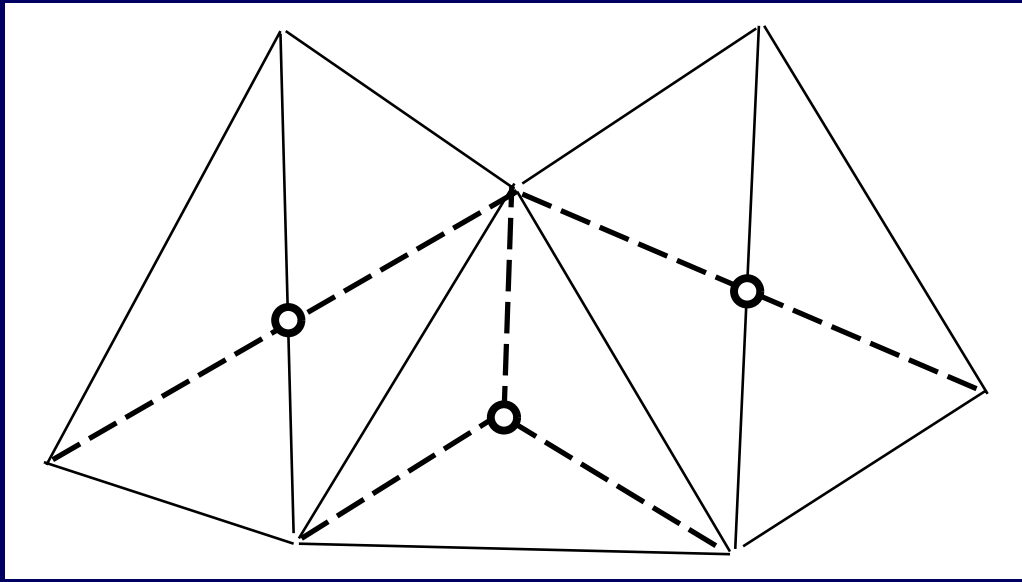
- Mesh Analysis



(based on longest edge)

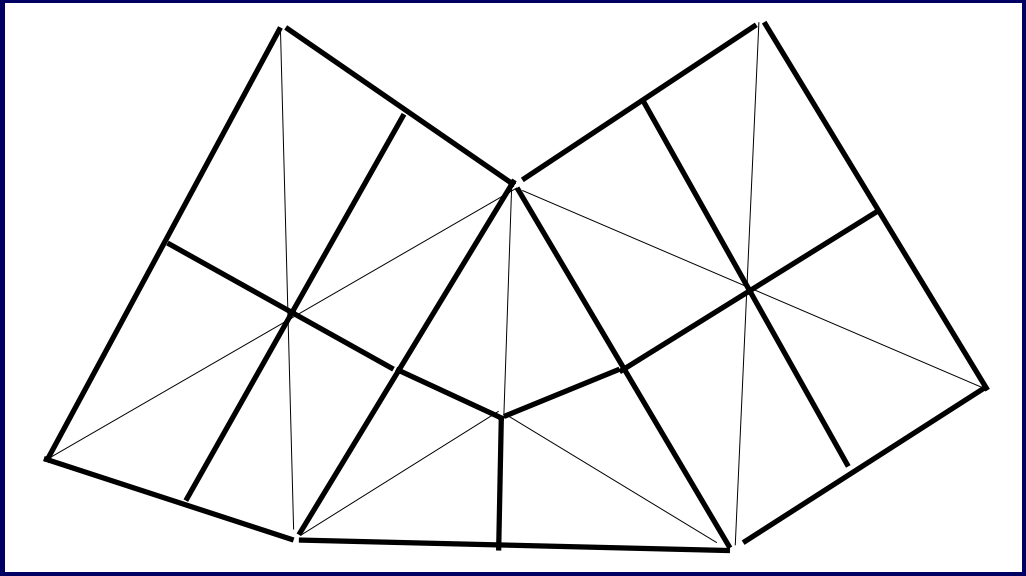
Step 2 - Hybrid Barycentric Subdivision

- Internal Block Structure



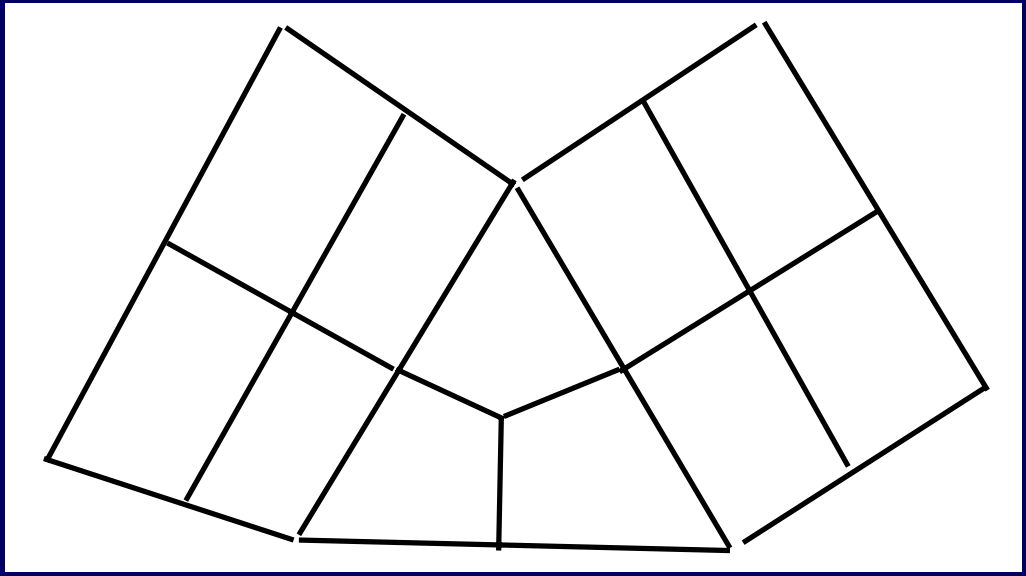
Step 3 - Final 4-8 Subdivision

- Boundary Matching



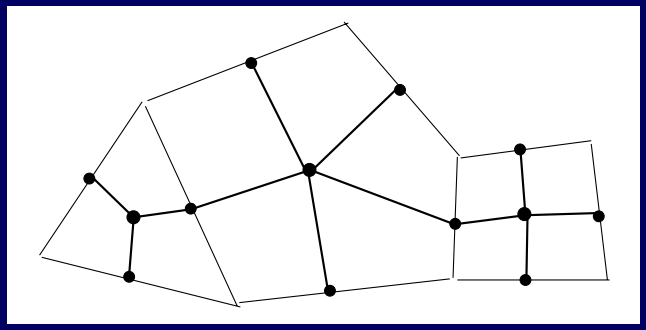
Step 4 - Tri-Quad Conversion

- Mesh Output



Discussion

- Relation with Catmull-Clark Subdivision



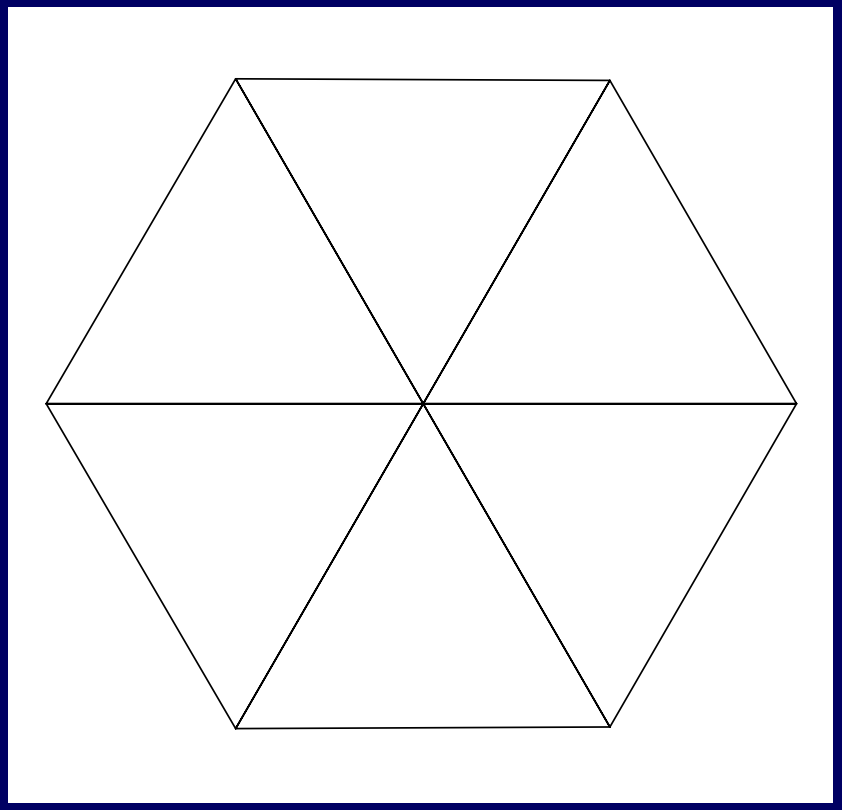
- Equivalent to:
  - Quad Identification
  - C-C Subdivision

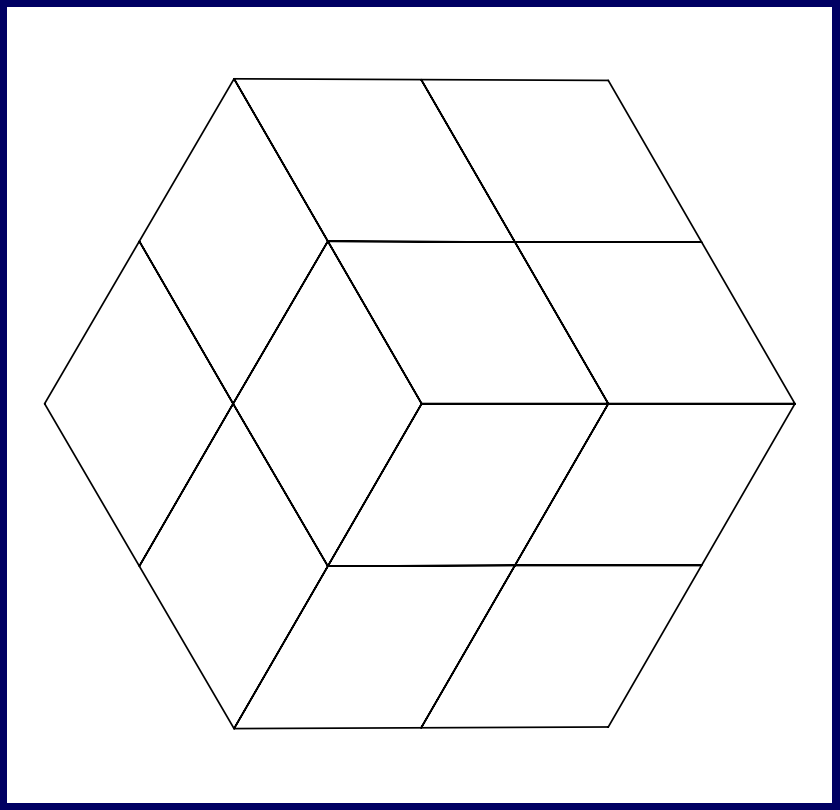
Results

- Examples
  - Planar Meshes
  - 3D Meshes

Hexagonal Disk

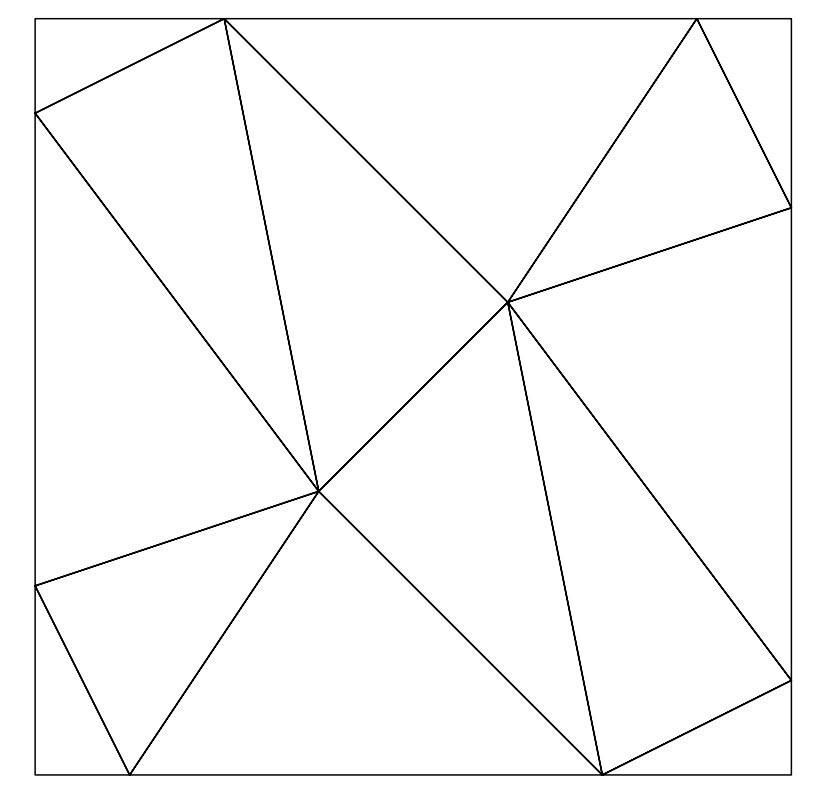
- Simple example (3 regions)

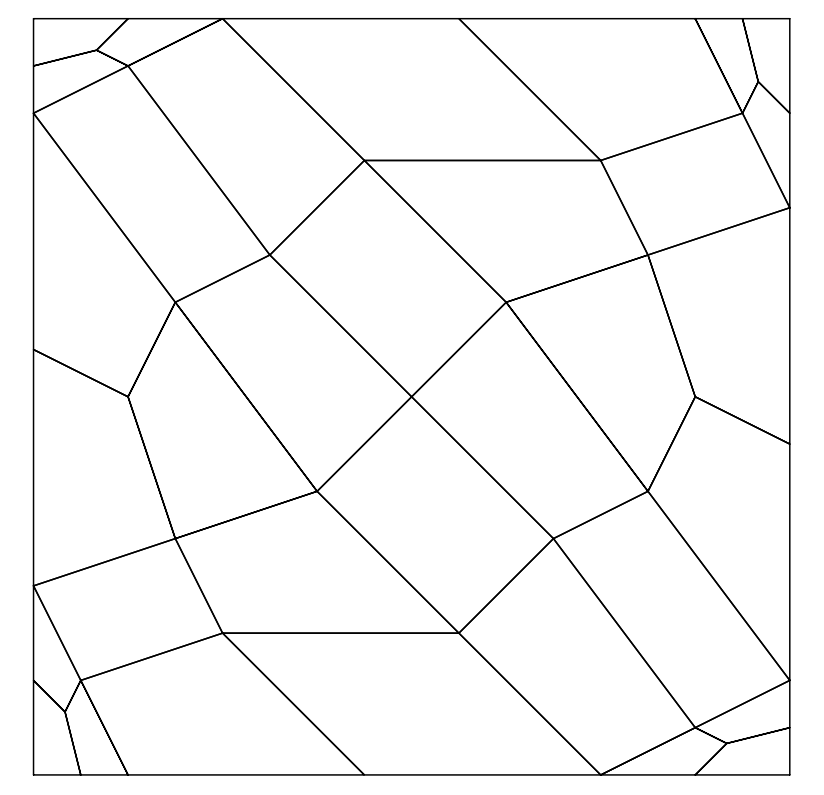




Coarse Mesh

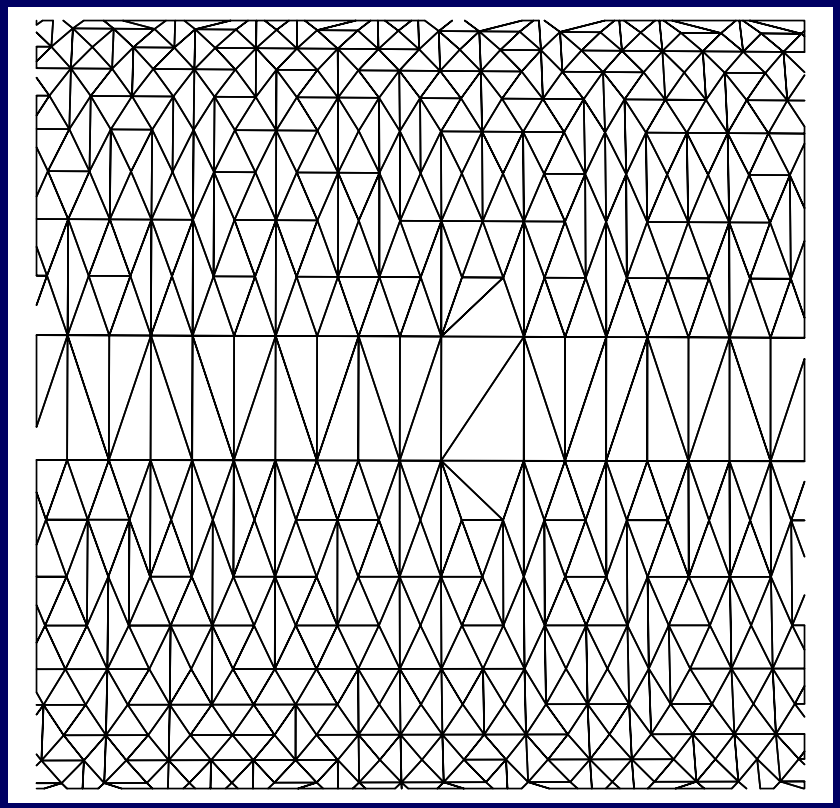
- Diagonal Directions

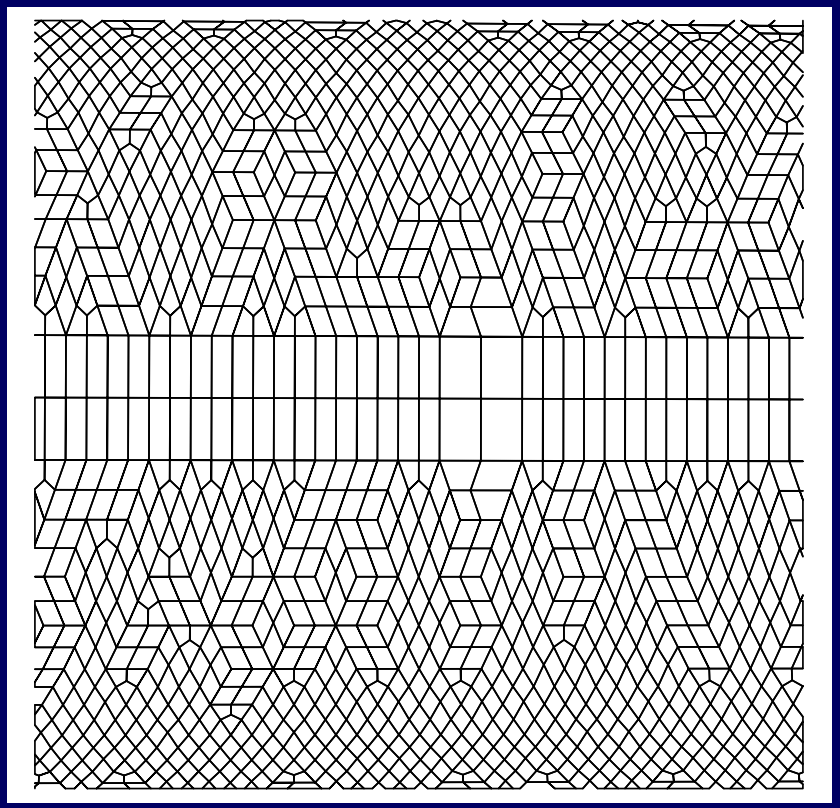




Fine Mesh

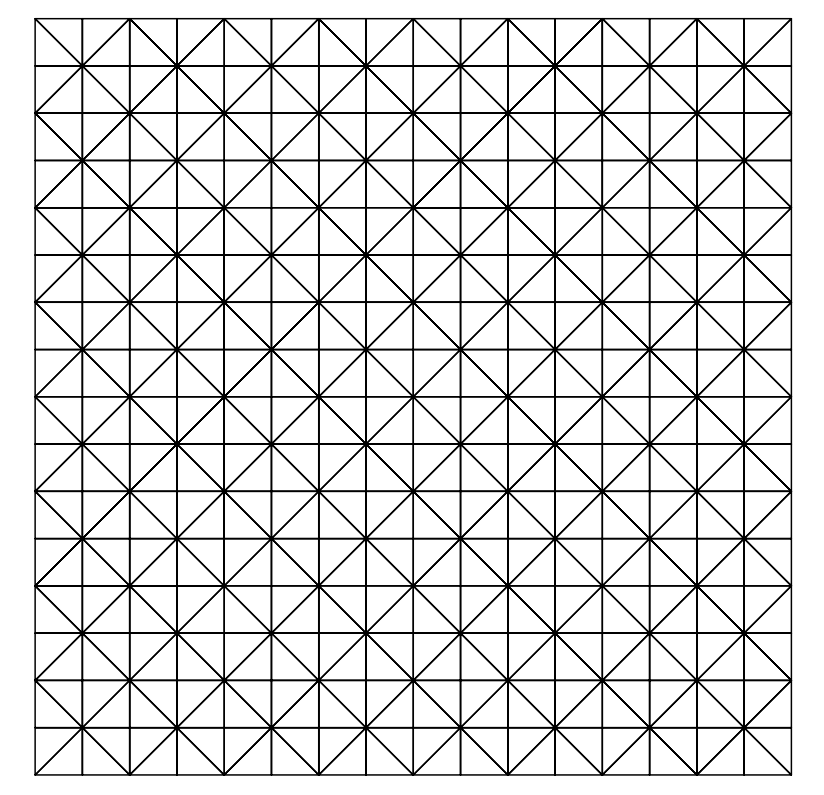
- Non-Uniform Triangulation (merged directions)

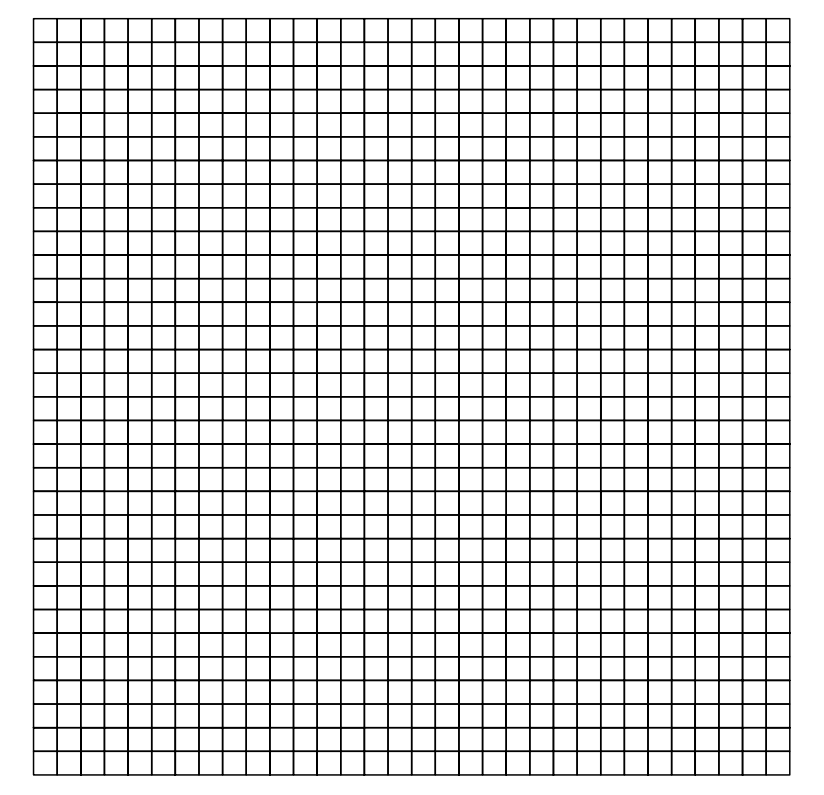




Four Directional Mesh

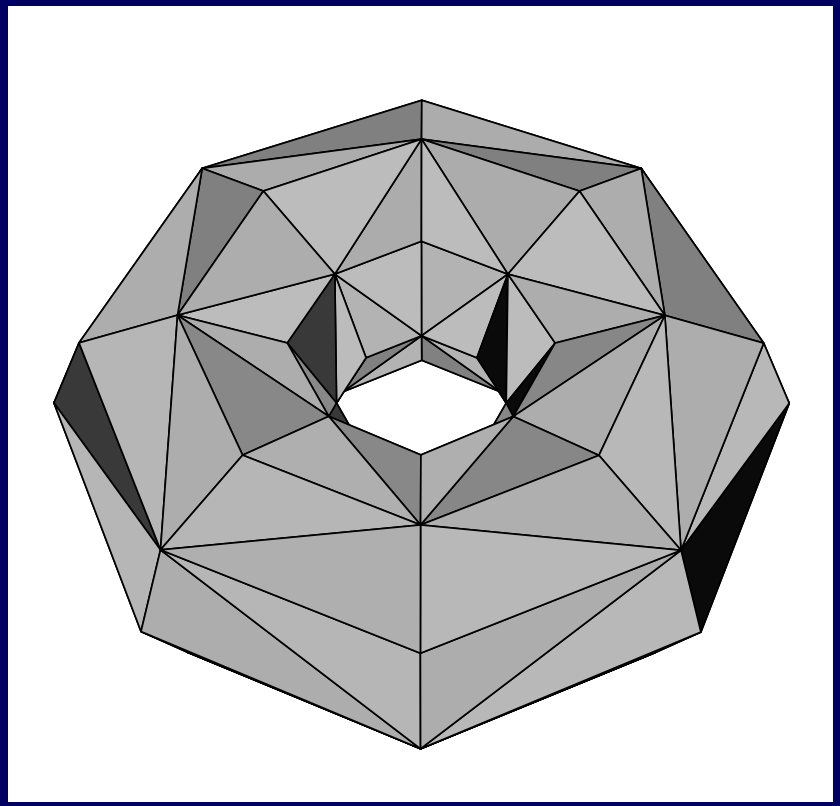
- Uniform Triangulation (recovered the grid)

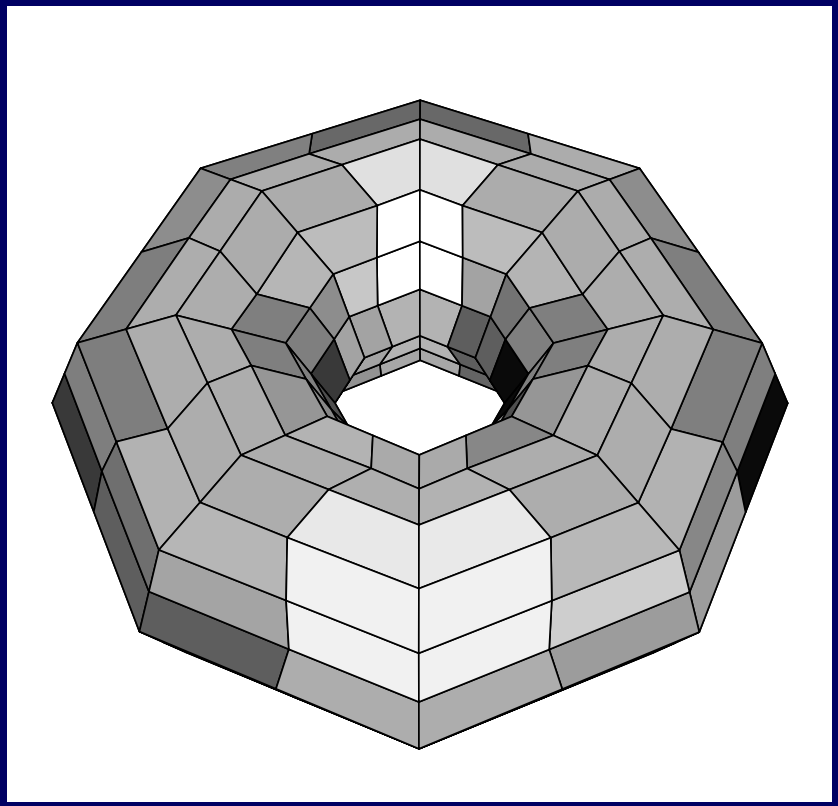




Torus

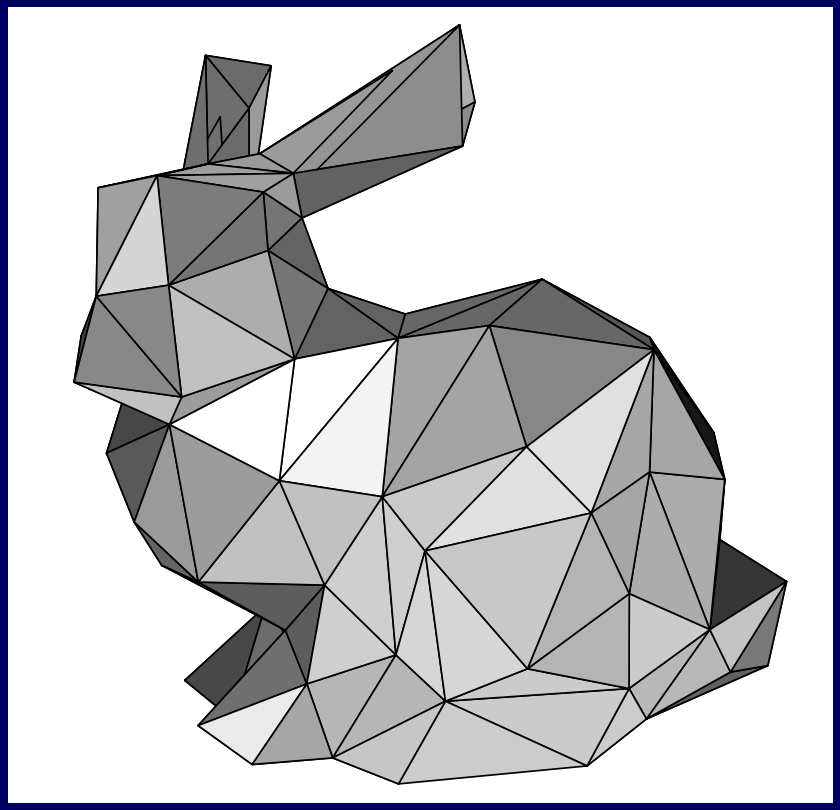
- 3D Mesh (parametric lines)

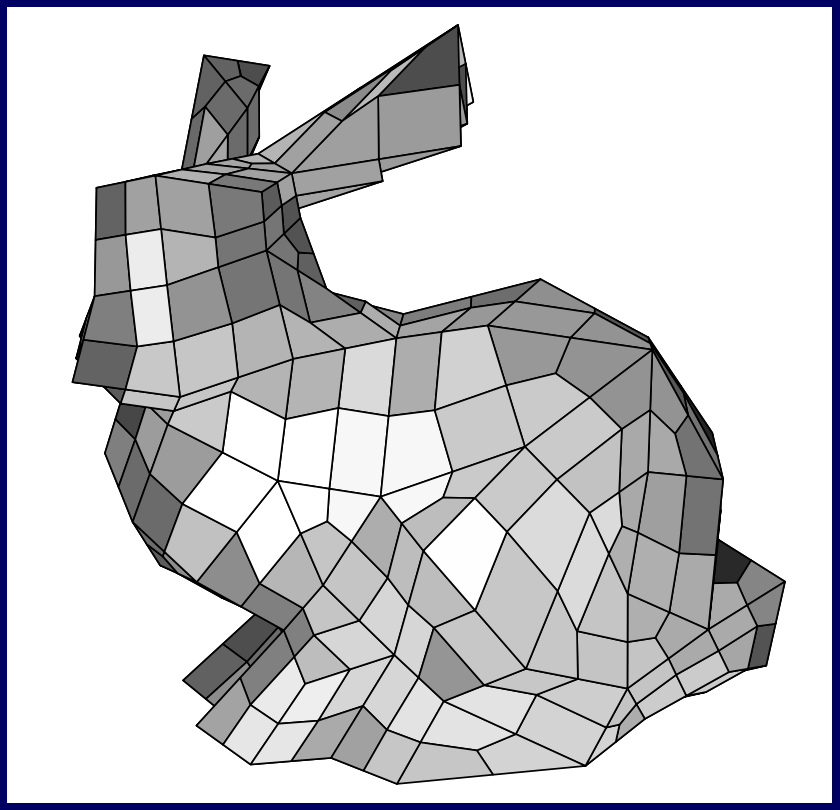




Stanford Bunny

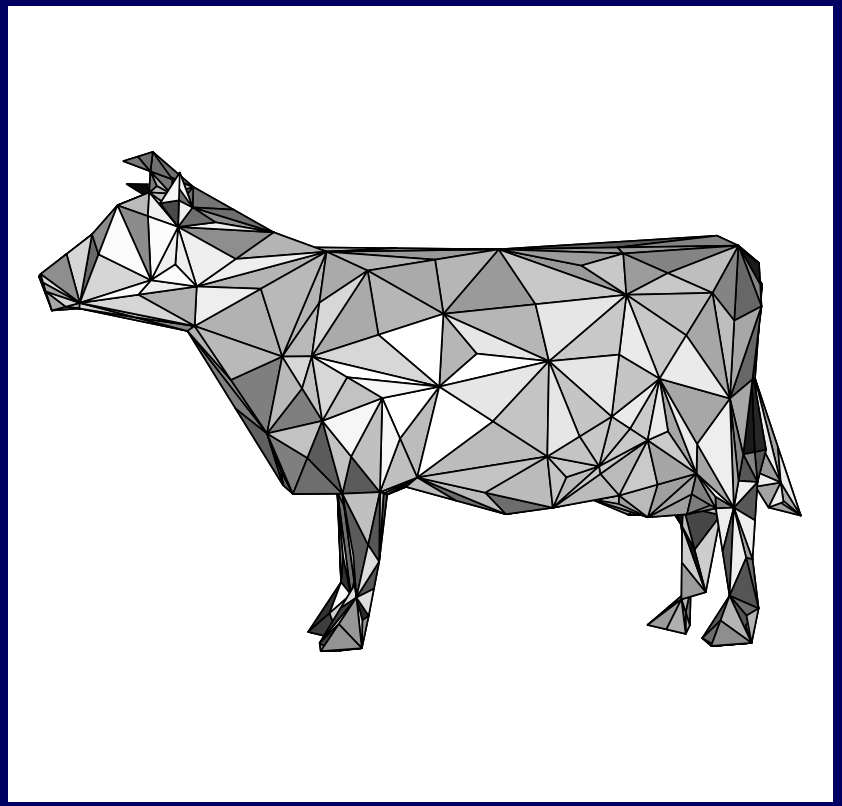
- Digitized 3D Object (uniform patches)

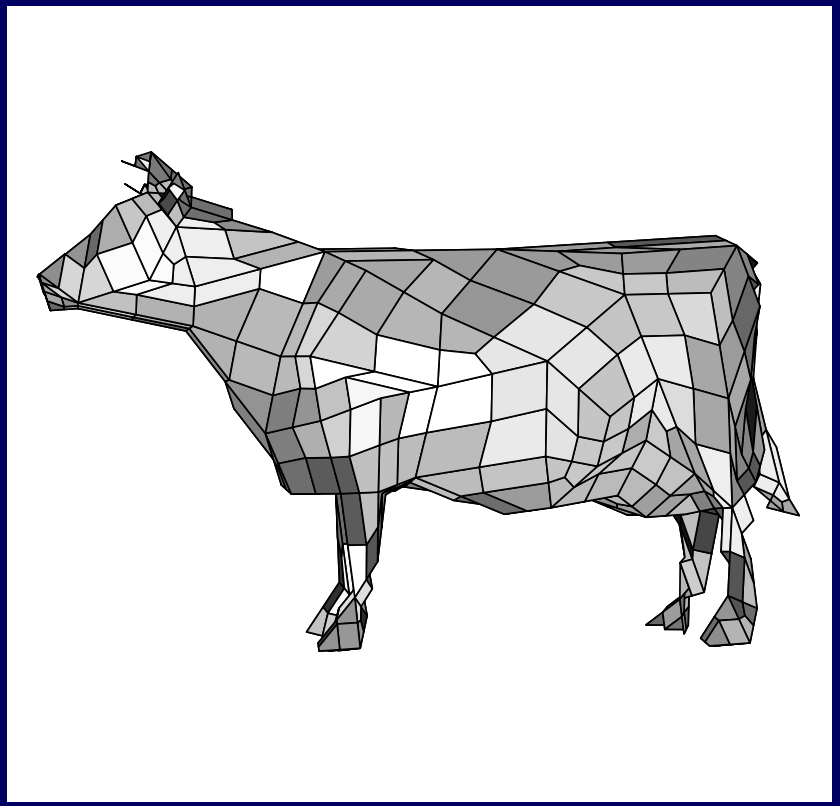




Viewpoint Cow

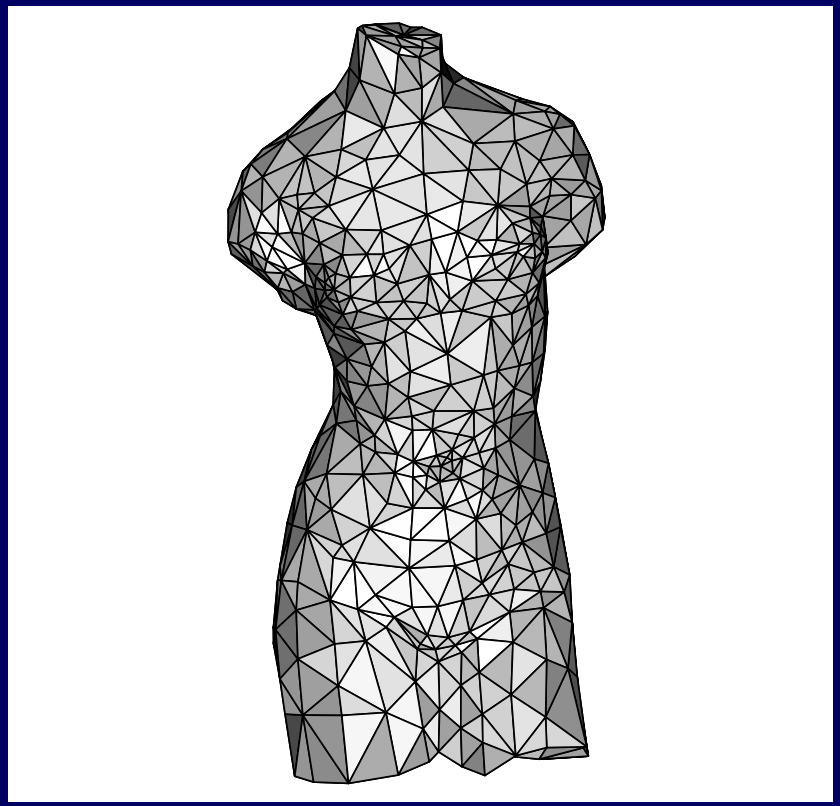
- Shape Library

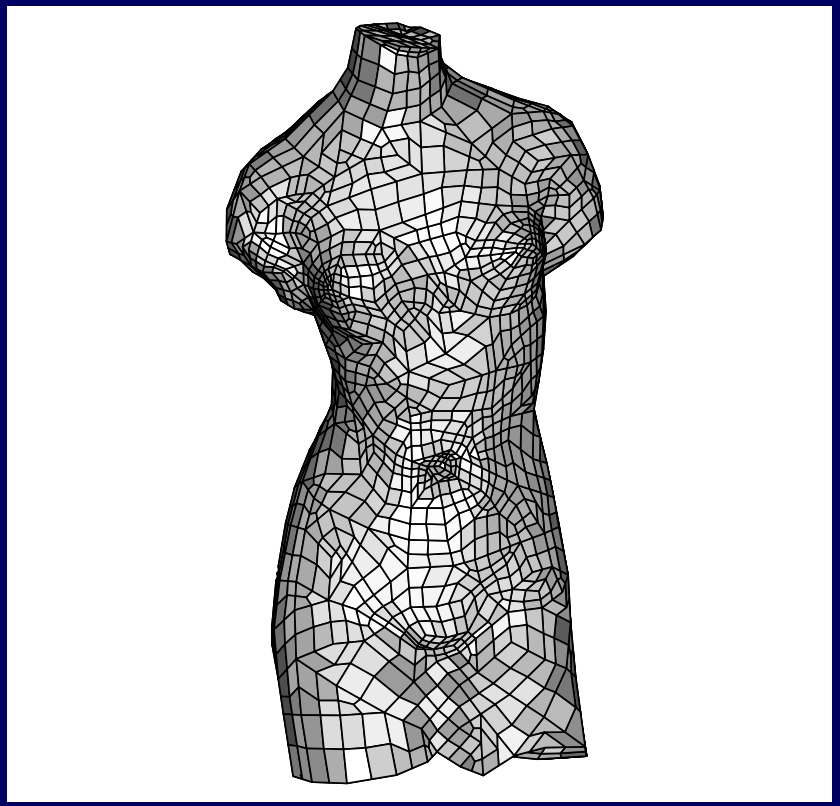




Venus

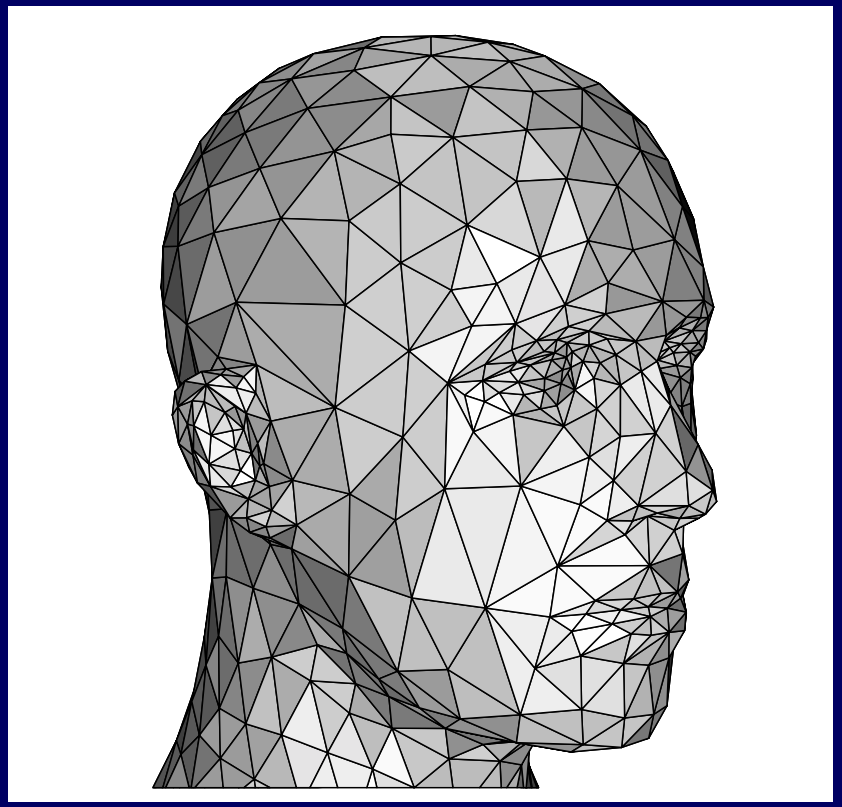
- (adaptation to shape)

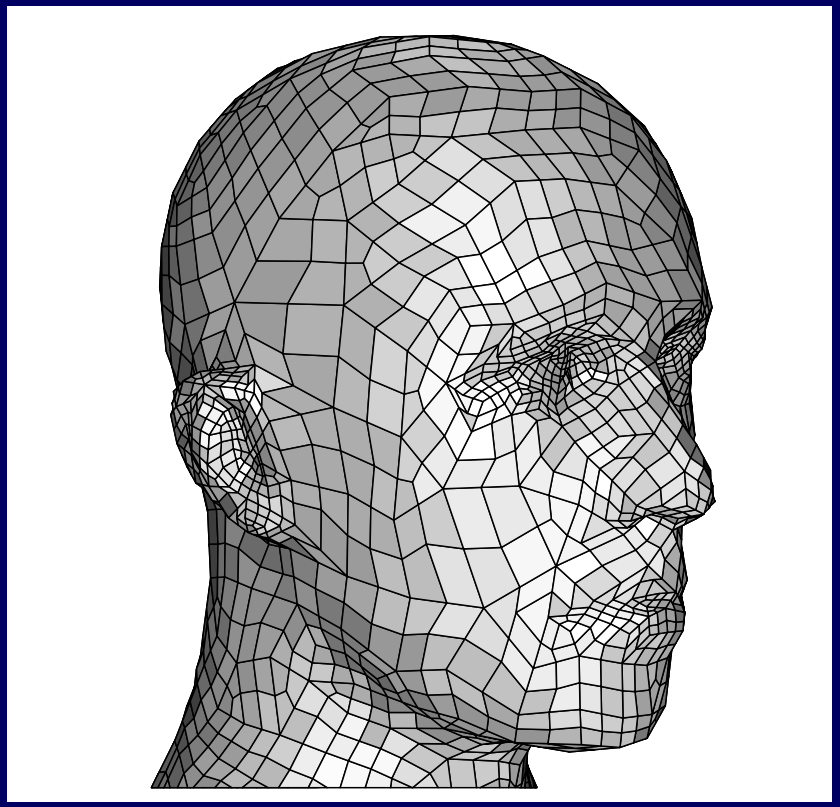




Mannequin Head - Quadrangulation

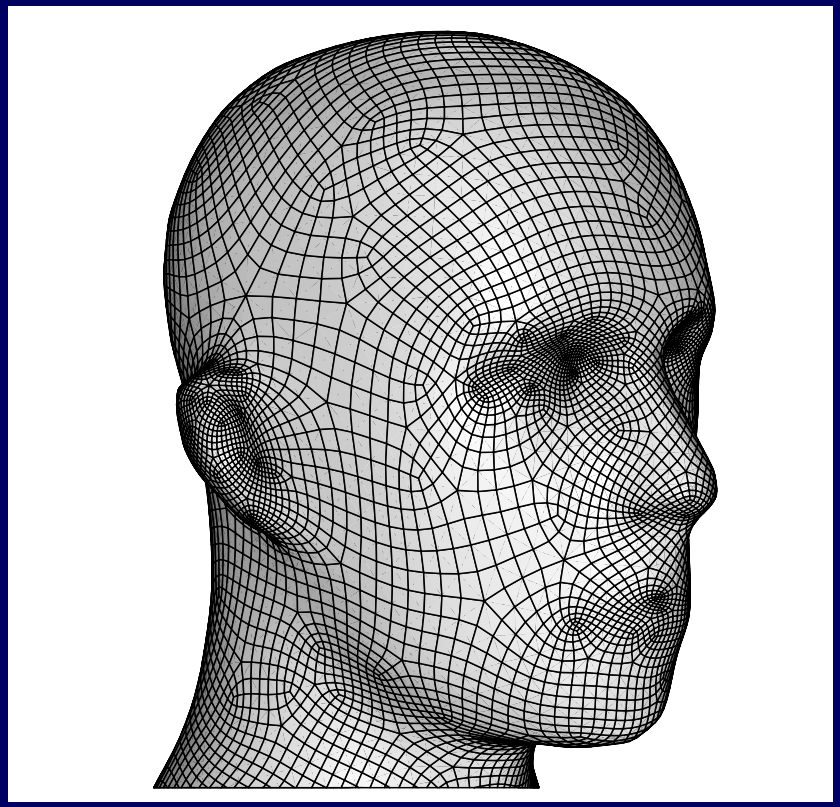
- Test Model for Subdivision

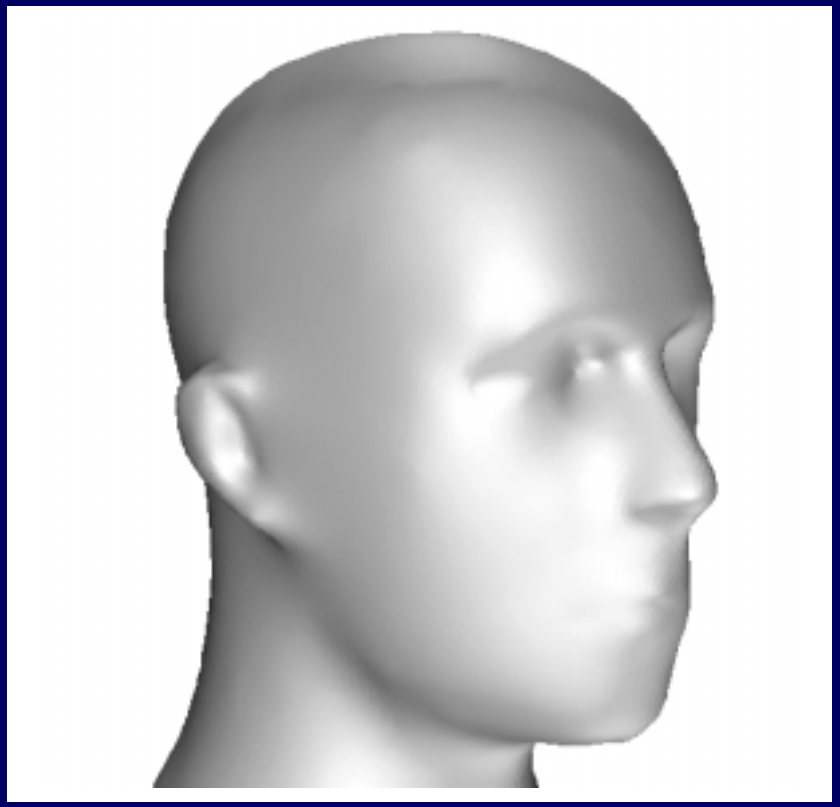




Mannequin Head - 4-8 Subdivision

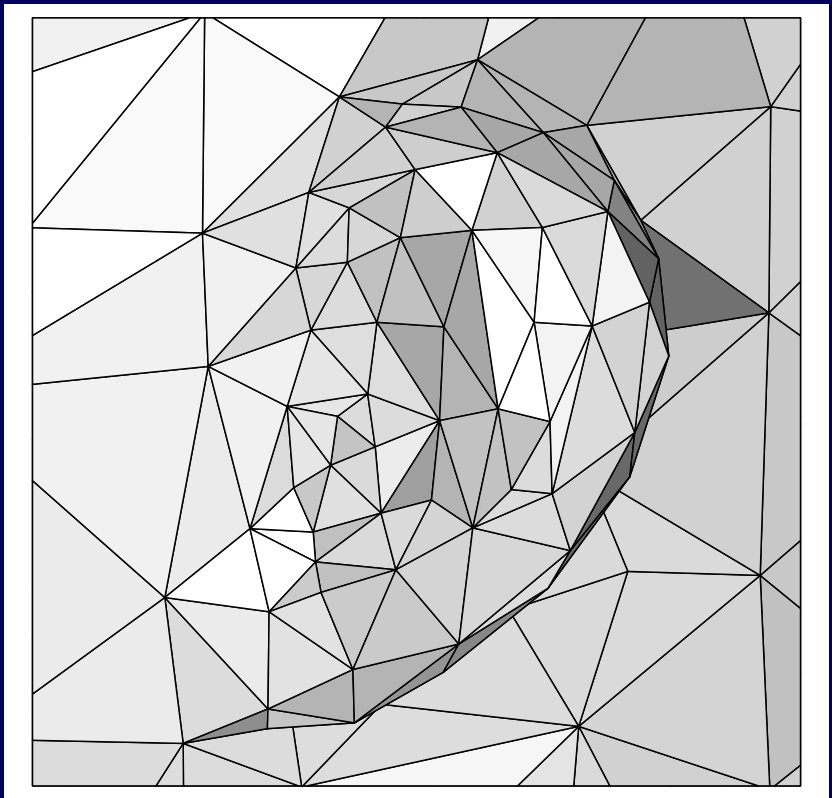
- 2 Levels of Subdivision

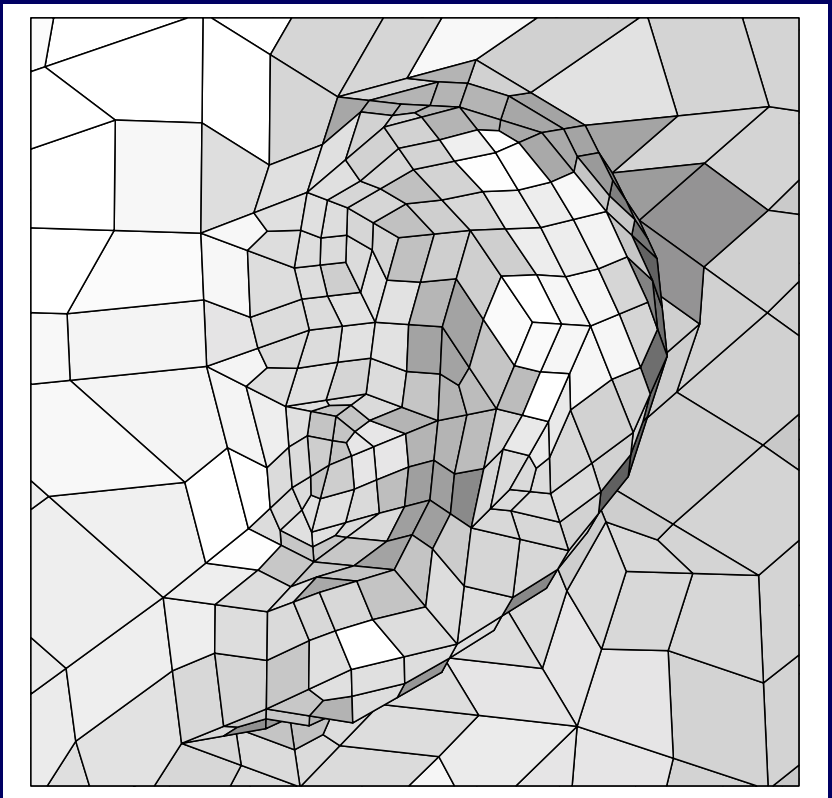




Ear - Quadrangulation

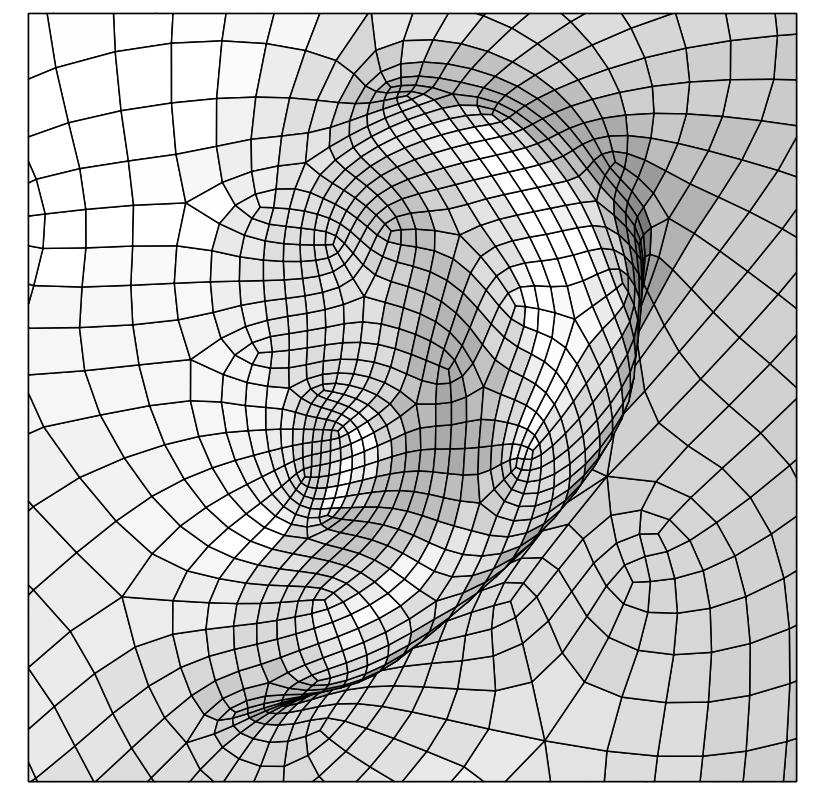
- Detail (transition)

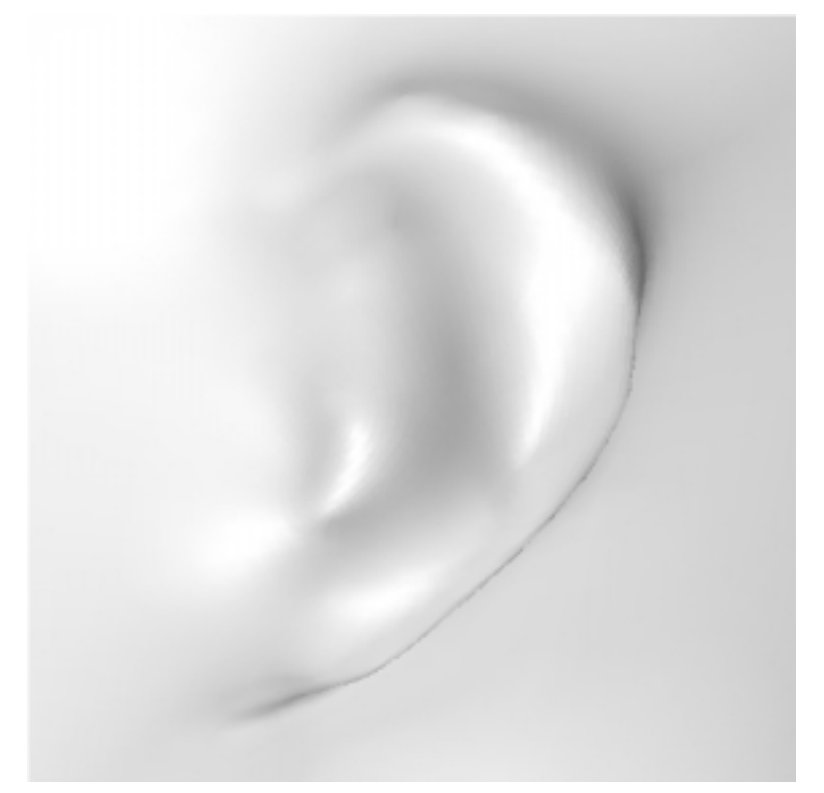




Ear (Detail) - 4-8 Subdivision

- Detail (smoothing)





Conclusions

- Simple, but Effective Approach
- Reasonable Quality Tessellation
- Moderate Mesh Size Increase
- Works well for Subdivision
- Can be used in other Applications