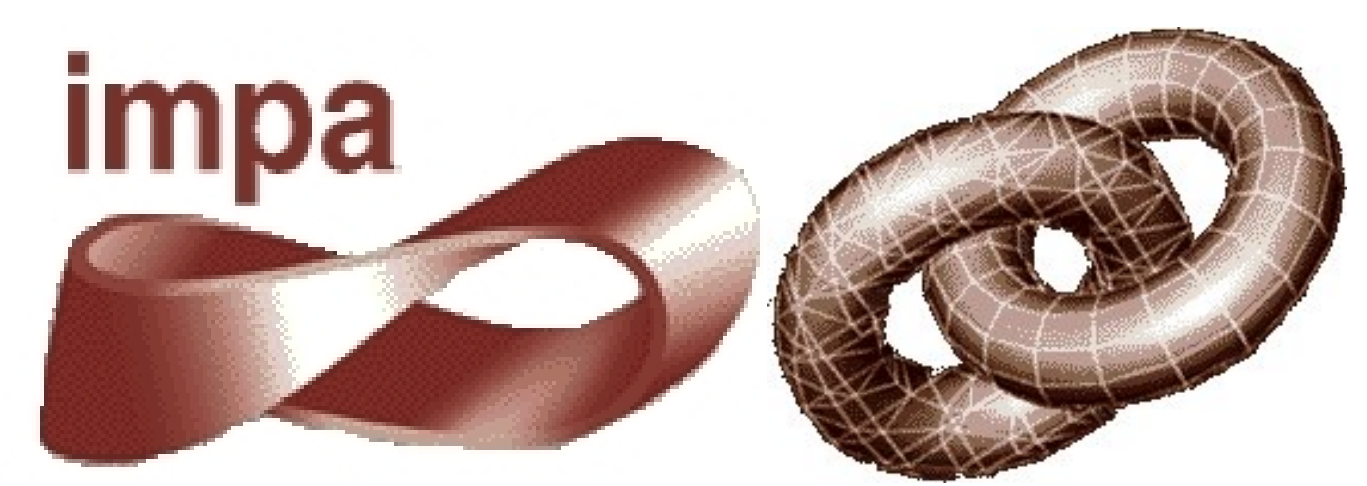


# Optimizing projections from the unit sphere to the plane for generation of panoramic images

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## Introduction

Panoramic images, i.e., images of wide fields of view, allow us to better represent an entire scene. The main difficulty in obtaining these images is to conciliate two important properties: preservation of object shapes and straight lines.

Our work is based on the work by Carroll et al. [1]. Our main contributions are the details about the optimization methods used in their method. All the statements in our work are proved in Sacht [2].

We represent the visible information of a scene from a viewpoint as a *viewing sphere*. Using longitude and latitude coordinates, we associate viewing spheres to images which we call *equirectangular images*. Examples of these images are the input images in the result section.

We state the panoramic image problem as that of finding a *projection*

$$\mathbf{u}: S \subseteq \mathbb{S}^2 \rightarrow \mathbb{R}^2 \\ (\lambda, \phi) \mapsto (u(\lambda, \phi), v(\lambda, \phi)) \quad ,$$

where  $S$  represents the FOV that will be projected.

## Distortion Energies

Carroll et al. [1] formulate energies that measure how a panoramic image contains undesirable distortions. These energies are related to preservation of shapes (conformality), bending of lines and smoothness of the projection. The final problem becomes alternating between minimizing

$$E_d = w_c^2 \|C\mathbf{x}\|^2 + w_s^2 \|S\mathbf{x}\|^2 + w_l^2 \|LO\mathbf{x}\|^2 + w_t^2 \|LDA\mathbf{x}\|^2$$

and

$$E_o = w_c^2 \|C\mathbf{x}\|^2 + w_s^2 \|S\mathbf{x}\|^2 + w_l^2 \|LO\mathbf{x}\|^2 + w_t^2 \|LOA\mathbf{x}\|^2,$$

where  $C, S, LO, LDA$  and  $LOA$  are matrices obtained from the modeling of the undesirable distortions and  $\mathbf{x}$  is a vector of unknown positions  $(u(\lambda, \phi), v(\lambda, \phi))$  for a uniform grid of the sphere. The alternation is performed until visual convergence is reached. We can rewrite both energies as

$$E_d = \|A_d\mathbf{x}\|^2 \text{ and } E_o = \|A_o\mathbf{x}\|^2$$

and the problem becomes minimizing energies of the form  $E(\mathbf{x}) = \|A\mathbf{x}\|^2$  at each step where  $A$  is a matrix.

## Minimization

We first consider the problem of minimizing  $E(\mathbf{x}) = \|A\mathbf{x}\|^2$  s.t.  $\|\mathbf{x}\| = 1$ . This choice of domain for minimization excludes the  $\mathbf{x} = \mathbf{0}$  solution and is convenient since scaled solutions represent same panoramic images.

**Statement 1:** The solution of  $\min_{\|\mathbf{x}\|=1} \|A\mathbf{x}\|^2$  is  $\mathbf{e}_1$ , the eigenvector of  $A^T A$  associated to its smallest eigenvalue  $\lambda_1$ . In addition,  $E(\mathbf{e}_1) = \lambda_1$ .

The following statement shows that constant mappings are eigenvectors of  $A^T A$  corresponding to  $\lambda = 0$ :

**Statement 2:** The vectors  $\mathbf{x}$  such that  $u_{ij} = K_u$  and  $v_{ij} = K_v, \forall i, j$ , are eigenvectors of  $A^T A$  with corresponding eigenvalues equal to 0.

We do not want constant mappings to be solutions of our optimization. Since the subspace of the projections in Statement 2 has dimension 2, we have to look for the eigenvector associated to the *third* smallest eigenvalue of  $A^T A$ .

An alternative is to minimize the perturbed energy

$$\tilde{E}(\mathbf{x}) = E(\mathbf{x}) + \varepsilon \|\mathbf{x} - \mathbf{y}\|^2 = \|A\mathbf{x}\|^2 + \varepsilon \|\mathbf{x} - \mathbf{y}\|^2,$$

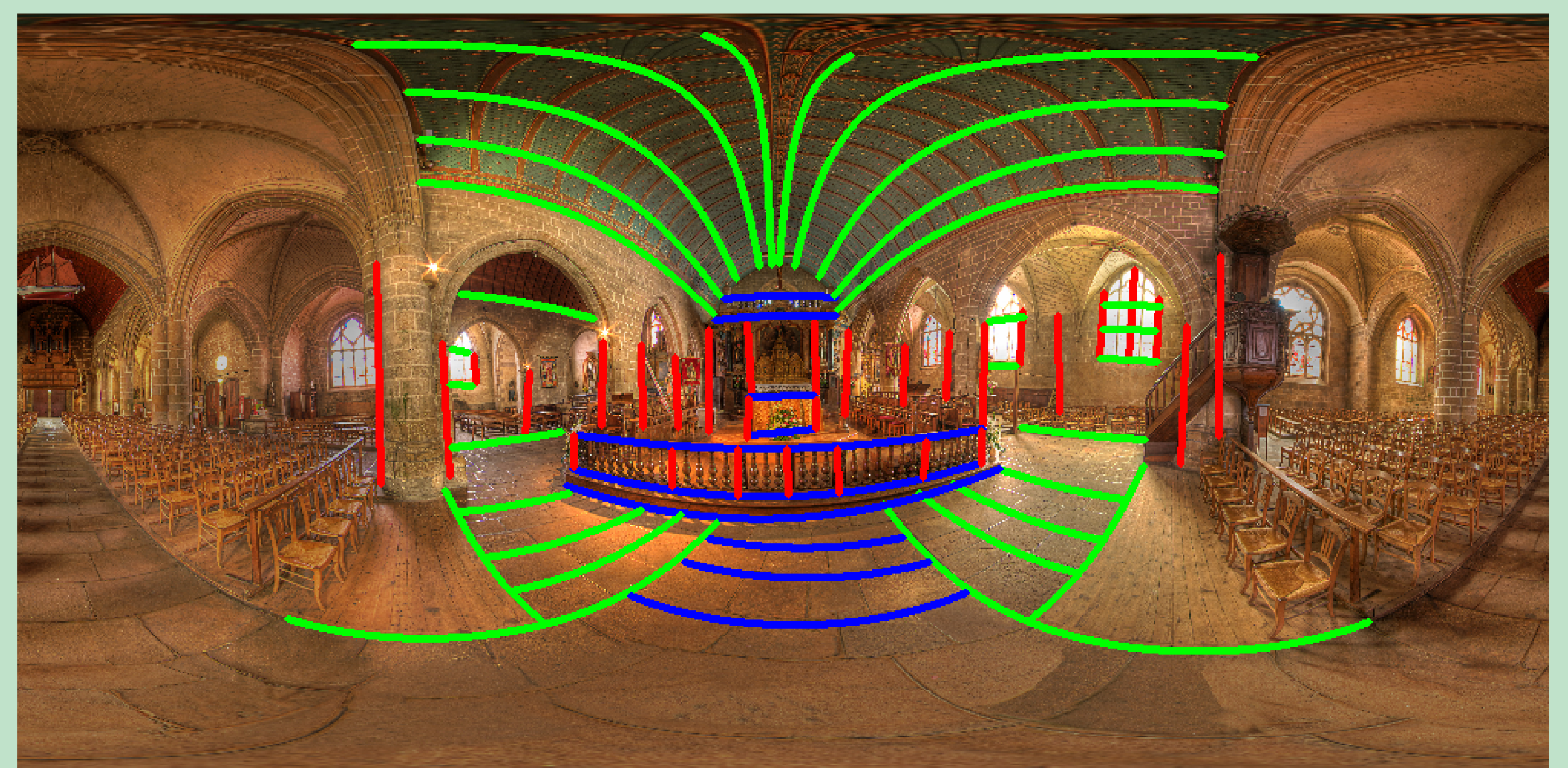
where  $\varepsilon = 10^{-6}$  and  $\mathbf{y}$  is some known discretized projection.

**Statement 3:** The minimizer of  $\tilde{E}$  on  $\mathbb{R}^n$  is  $\mathbf{x} = (A^T A + \varepsilon I)^{-1}(\varepsilon \mathbf{y})$ .

The main advantage of minimizing  $\tilde{E}$  instead of minimizing  $E$  is that we replace an eigenvalue problem by solving a linear system. Furthermore,  $A^T A + \varepsilon I$  is sparse, symmetric and positive definite.

## Results

In this section, we show some results produced by our implementation of the method. The marked lines are used to formulate the line energies mentioned in the previous sections.



## Acknowledgements

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## References

- [1] R. Carroll, M. Agrawala, A. Agarwala. Optimizing Content-Preserving Projections for Wide-Angle Images. In *Siggraph 2009*.
- [2] L.K. Sacht. Content-Based Projections for Panoramic Images and Videos. Master's Thesis, IMPA, 2010.