

Optimizing projections from the unit sphere to the plane for generation of panoramic images

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ABSTRACT

Introduction: Common cameras capture a limited field of view, usually of up to ninety degrees. The reason for this fact is that when the field of view (FOV) becomes wider, the projection used by these cameras introduces strong distortions in the objects.

Panoramic images, which are images of wide FOVs, can better represent an entire scene.

Our work is based in Carroll et al. [1], who suggested all the distortions we consider. The contributions of this abstract are the details about the optimization methods, which were briefly mentioned in [1]. Almost all details that will be exposed here were not treated in this reference.

We represent the visible information of a scene seen from a viewpoint at a given moment as the *viewing sphere*, which is the unit sphere where each point has an associated color, the color that is seen when one looks towards the point. Using longitude and latitude coordinates $((\lambda, \phi) \in [-\pi, \pi] \times [-\frac{\pi}{2}, \frac{\pi}{2}])$ we associate the viewing sphere to images which we call *equirectangular images* (many of these images can be found in photo sharing sites, Flickr [2], for example). With these notations, we state the problem of finding a panoramic image as that of finding a *projection*

$$\begin{aligned} \mathbf{u} : S \subseteq \mathbb{S}^2 &\rightarrow \mathbb{R}^2 \\ (\lambda, \phi) &\mapsto (u(\lambda, \phi), v(\lambda, \phi)) \end{aligned}$$

where S is a subset of the viewing sphere, the FOV that is going to be projected.

Distortion energies and optimization: Carroll et al. [1] formulate energies that measure how a panoramic image contains undesirable distortions. These energies are related to preservation of object shapes (conformality), bending of lines and smoothness of the projection. For additional details, we also refer the reader to Sacht [3]. The final problem becomes alternating between minimizing

$$E_d = w_c^2 \|C\mathbf{x}\|^2 + w_s^2 \|S\mathbf{x}\|^2 + w_l^2 \|LO\mathbf{x}\|^2 + w_l^2 \|LDA\mathbf{x}\|^2$$

and

$$E_o = w_c^2 \|C\mathbf{x}\|^2 + w_s^2 \|S\mathbf{x}\|^2 + w_l^2 \|LO\mathbf{x}\|^2 + w_l^2 \|LOA\mathbf{x}\|^2,$$

where C , S , LO , LDA and LOA are matrices obtained from the modeling of the undesirable distortions and \mathbf{x} is a vector of unknown positions $(u(\lambda, \phi), v(\lambda, \phi))$ for a uniform grid of the sphere. For our implementation, we set $w_c = 0.4$, $w_s = 0.05$ and $w_l = 1000$. The alternation is performed until visual convergence is reached. We can rewrite both energies as

$$E_d = \|A_d\mathbf{x}\|^2 \text{ and } E_o = \|A_o\mathbf{x}\|^2$$

and the problem becomes minimizing energies of the form $E(\mathbf{x}) = \|A\mathbf{x}\|^2$ at each step. It is obvious that if we consider the unconstrained problem of minimizing E in \mathbb{R}^n , $\mathbf{x} = \mathbf{0}$ will be a solution. However, this solution represents all points of the sphere being mapped to the origin.

We consider the problem of minimizing $E(\mathbf{x})$ subject to $\|\mathbf{x}\| = 1$. The solution of this problem is \mathbf{e}_1 , the eigenvector of $A^T A$ associated to its smallest eigenvalue. For the case of the energies we are considering, an additional problem arises: the eigenvectors of $A^T A$ associated to the two smallest eigenvalues also represent trivial projections. Thus, we have to look for the third eigenvector of $A^T A$.

Since this becomes untractable due to the size of our problem (thousands of variables) we propose to minimize the perturbed energy

$$\tilde{E}(\mathbf{x}) = E(\mathbf{x}) + \varepsilon \|\mathbf{x} - \mathbf{y}\|^2 = \|A\mathbf{x}\|^2 + \varepsilon \|\mathbf{x} - \mathbf{y}\|^2,$$

where $\varepsilon = 10^{-6}$ and \mathbf{y} is some known discretized projection (we use the stereographic projection).

Considering it as an unconstrained problem we show [3] that the minimizer is given by

$$\mathbf{x} = (A^T A + \varepsilon I)^{-1}(\varepsilon \mathbf{y}).$$

By using this minimizer as solution for the problem, we replace an eigenvalue problem by solving a linear system where the matrix is sparse, symmetric and positive definite, which is convenient for numerical methods.

Results: In figure 1, we show results produced by our implementation of the method. For implementation details, we refer the reader to Sacht [3].

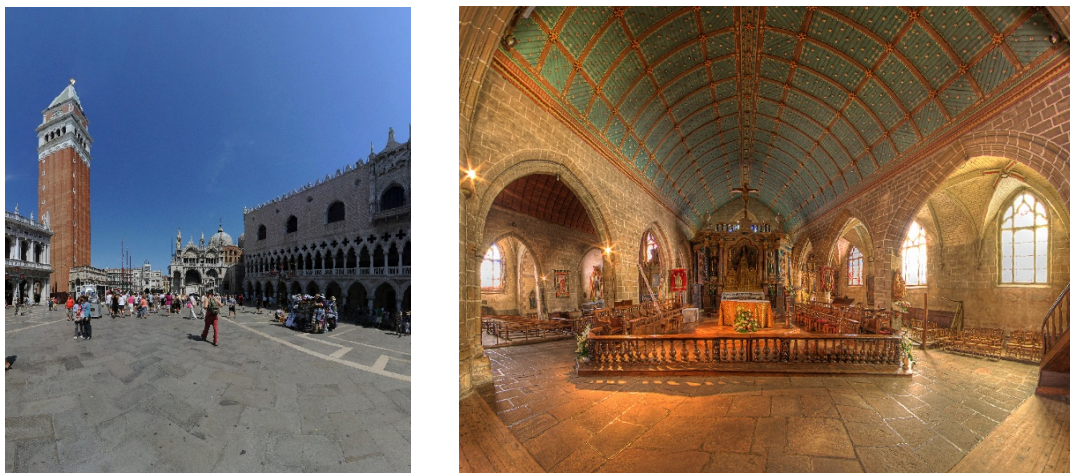


Figure 1: Results produced by the method studied in this abstract. One may observe that the straight lines in the scene and the shapes of the objects are well preserved. Left: 180 degree longitude/160 degree latitude FOV. Right: 210 degree longitude/130 degree latitude FOV.

Future Work: We intend to extend this approach to produce panoramic videos, which are videos where each frame is constructed from a wide FOV. Applications in many areas as cinema and sport broadcasting are evident. Some initial work in this topic may be found in Sacht [3].

Keywords: *Panoramic images, image processing, optimization.*

References

- [1] R. Carroll, M. Agrawal, A. Agrawala, Optimizing content-preserving projections for wide-angle images, *ACM Trans. Graph.*, vol. 28, no.3 (2009) 1-9.
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