17 August 2020. Lecture 1: Historical Introduction.

Section 2.1, 2.2

$P(x) \in \mathbb{R}[x]$ = ring of polynomials with coefs in $\mathbb{R}$ and in a single varible

$$P(x) = (x-t_1)(x-t_2)(x-t_3), \quad t_1, t_2, t_3 \in \mathbb{R}$$

$$t_1 < t_2 < t_3$$

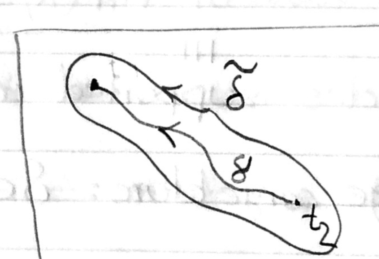
Elliptic integral $\qquad \displaystyle\int_{t_1}^{t_2} \frac{dx}{\sqrt{P(x)}}$

1800–1900: <u>One cannot compute integrals</u>

Explain Exercise 2.2, 2.3, 2.4, 2.5

Complexification: $t_1, t_2, t_3 \in \mathbb{C}$, $P(x) \in \mathbb{C}[x]$

$$2\int_{\delta} \frac{dx}{\sqrt{P(x)}} = \int_{\tilde{\delta}} \frac{dx}{\sqrt{P(x)}}$$



$\mathbb{C}$

For a polynomial of arbitrary degree, we use Pochhamner cycle insted of δ, see Ex. 4.7

The variable $y = \sqrt{P(x)}$

Elliptic curves $\qquad E := \{(x,y) \in \mathbb{C}^2 \mid y^2 = P(x)\}$

we will see in the course that $\quad \overset{C^\infty}{\cong}$

Figure 2.1. $\tilde{\delta}$ can be lift up to a closed path in E $\quad$ Removed point

$\hat{\delta}$

we arrive at $\displaystyle\int_\delta \frac{dx}{y}$, $\delta \in H_1(E,\mathbb{Z})$

In general, we have "periods"

$$\int_\delta \omega \quad , \quad \delta \in H_n(X,\mathbb{Z}), \quad \omega \in H^n_{dR}(X)$$

X a smooth projective variety $/\mathbb{C}$.

We are interested in the Hodge conjecture

$Z_i \subseteq X$ a subvariety of $\dim_{\mathbb{C}} \frac{n}{2}$ for n even

$$[Z] \in H_n(X,\mathbb{Z})$$

$Z \rightsquigarrow$ Zyklus

Hodge conjecture gives us a criterion to classify the $\mathbb{Z}$-submodule of $H_n(X,\mathbb{Z})$ generated by $[Z]$'s.

$\parallel\!|$

cycles supported in algebraic cycles

Hodge conjecture: $\delta \in H_n(X,\mathbb{Z})$ is supported in algebraic cycles $\Longleftrightarrow$ certain periods $\int_\delta \omega$, $\omega \in H^n_{dR}(X)$ vanishes

Hodge Filtration.