HOMEWORK 4

1. **Exercise.** Prove that (V, Δ_{F_4}) as in class is an indecomposable root system.

2. **Exercise.** Describe an embedding of $\mathfrak{sl}_2 \hookrightarrow \mathfrak{so}_8 =: V$. Consider the corresponding $\mathfrak{sl}_2$-module V and describe its decomposition in irreducible modules.

3. **Exercise.** Work out all the Cartan matrices and Dynkin diagrams for the root systems $A-G$. Do not write this down cause no-one will read that.

4. **Exercise.** Let A be an abstract Cartan matrix. Show that there exists a diagonal matrix D with positive entries such that DAD^{-1} is symmetric and positive definite.

5. **Exercise.** Let A be a positive definite symmetric matrix with ones in the diagonal. Show that there exists a positive definite square root $A^{1/2}$.

6. **Exercise.** Let A be an abstract Cartan Matrix and D be its associated Dynkin diagram. Suppose there are two vertices i and j that are joined by a single edge and let D' denote the graph obtained by D by removing the edge and collapsing the two vertices. Show that D' is the Dinkin diagram of an abstract Cartan matrix A'.

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Date: Due Tuesday Oct 21.