INTRODUCTION TO LIE ALGEBRAS
PROBLEM SET 2

IMPA

Exercise 1. *Let* $I \subset \mathfrak{g}$ *be an ideal. Prove that any member of the derived and central sequence of* I *is also an ideal of* $\mathfrak{g}$.

Exercise 2. *Prove that* $\mathfrak{sl}_2(\mathbb{F})$ *is Nilpotent if the characteristic of* $\mathbb{F}$ *is* 2.

Exercise 3. *Let* $A, B \in \mathfrak{gl}(V)$ *be such that* $[A, B] = 0$. *Show that* $(A + B)_s = A_s + B_s$ *and* $(A + B)_n = A_n + B_n$. *Show first that if* A, B *are semisimple (resp. nilpotent) then* $A + B$ *is semisimple (resp. nilpotent).*

Exercise 4. *Let* $A, B, C \in \mathfrak{gl}(V)$, *check that* $\mathrm{tr}([A, B]C) = \mathrm{tr}(A[B, C])$.

Exercise 5. *Let* $\mathfrak{g} \subset \mathfrak{gl}(V)$ *be a solvable Lie subalgebra. Show that* $\mathrm{tr}(AB) = 0, \forall A \in [\mathfrak{g}, \mathfrak{g}], B \in \mathfrak{g}$.

Exercise 6. *Prove that the Killing form of a Nilpotent Lie algebra is zero.*

Exercise 7. *Consider the standard basis of* $\mathfrak{sl}_2(\mathbb{F})$. *Compute its dual with respect to the Killing form.*

Exercise 8. *If* A *and* B *are commuting nilpotent operators, show that* $e^{A+B} = e^A e^B$.

Date: Due Sept 4th 2014.