Homework 6

Due 24/5/2018*

1 Exercise. Show that 29 is not prime in $\mathbb{Z}[\sqrt{-5}]$.

2 Exercise. Show that in a principal ideal domain given $a, b \in R$, their gcd d exists and that there exists $p, q \in R$ such that $pa + qb = d$. Find a counterexample for the last statement in an UFD.

3 Exercise. Define and prove existence and uniqueness (modulo units) of the *least common multiple* in a PID.

4 Exercise. prove that $xw - zy$ is irreducible in $\mathbb{C}[x, y, z, w]$.

5 Exercise. Let $f \in \mathbb{C}[x, y]$ be irreducible and suppose that the variety $V(g)$ of another polynomial g contains $V(f)$. Show that f divides g.

6 Exercise. Prove that two integer polynomials are relatively prime in $\mathbb{Q}[x]$ if an only if the ideal they generate in $\mathbb{Z}[x]$ contains an integer.

7 Exercise. Find a ring R (not an integral domain) and elements $a, b \in R$ that are associate and do not differ by a unit. (This was stated incorrectly in class!).

*Starred exercises are optional