Homework 5

Due 15/5/2018*

All rings are commutative.

1 Exercise. Determine all rings that contain the zero ring as a subring.

2 Exercise. Prove that if $R \simeq R'$ then $R[x] \simeq R'[x]$. Can you relate the group of automorphisms of R with that of $R[x]$?

3 Exercise. determine all ideals of the ring $\mathbb{R}[[t]]$.

4 Exercise. Let F be a field of characteristic different than 2. Show that $F[x]/(x^2)$ is not isomorphic to $F[x]/(x^2 - 1)$.

5 Exercise. Let R be an integral domain, check that $R[x]$ is an integral domain

6 Exercise. Let R be a ring and $I \subset R$ an ideal such that every element of $R \setminus I$ is a unit of R. Prove that I is a maximal ideal and that R has no other maximal ideal. Can you find such an example where moreover there are other ideals different than $0, I$ or R?

7 Exercise. Find a maximal ideal of $\mathbb{R}[x, y]$ that is not generated by $(x - a, y - b)$ for $a, b \in \mathbb{R}$.

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*Starred exercises are optional