

# Homework 1

Due 20/3/2018

**1 Exercise.** Let  $S$  be a set with an associative binary map and with an identity element. Prove that the subset of  $S$  consisting of invertible elements is a group. Find a counterexample for the set of *left invertible* elements.

**2 Exercise.** Let  $(G, \cdot)$  be a group. Define the group  $G^{op}$  as the set  $G$  but with the operation  $a \cdot^{op} b := b \cdot a$ . Show that  $G^{op}$  is a group.

**3 Exercise.** Let  $G$  be a cyclic group of order  $n$  and let  $r|n$ . Prove that  $G$  contains exactly one subgroup of order  $r$ .

**4 Exercise.** Let  $SO_3(\mathbb{R})$  be the set of  $3 \times 3$  orthogonal real matrices with determinant 1. Prove that it is a subgroup of  $GL_3(\mathbb{R})$ .

**5 Exercise.** Let  $\Delta$  be the regular tetrahedron in  $\mathbb{R}^3$  so that its four vertices are in the sphere of radius 1 centered at the origin. Let

$$G = \{A \in SO_3(\mathbb{R}) \mid A\Delta = \Delta\}.$$

Show that  $G$  is a subgroup of  $SO_3(\mathbb{R})$  isomorphic to  $A_4$ , the alternating group of permutations of 4 elements.

**6 Exercise.** Prove that the matrices

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

are conjugated in  $GL_2(\mathbb{R})$  but are not conjugated in  $SL_2(\mathbb{R})$ .

**7 Exercise.** Let  $\phi, \psi : G \rightarrow G'$  be two morphisms between groups and let  $H \subset G$  consists of elements  $g \in G$  such that  $\psi(g) = \phi(g)$ . Is  $H$  a subgroup of  $G$ ?

**8 Exercise.** Find an  $\mathbb{N} \times \mathbb{N}$  matrix of natural numbers such that in each row every natural number appears and there are no repetitions and in each column every natural number appears and there are no repetitions.