A few exercises more

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Please try solving these exercises without looking their solutions up. The ones marked with one skull (💀) are harder. The ones marked with a skull and a question mark (💀?) may or may not be hard (Let me know.)

1. 💀 Is there a “color translation function” $m$ that allows us to convert between the results of alpha blending in gamma and linear spaces? I.e., is there $m$ such that, for all linear $f, \alpha_f$ and $b, \alpha_b$, we have

$$
\gamma(m(f, \alpha_f) \oplus m(b, \alpha_b)) = \gamma(f, \alpha_f) \oplus \gamma(b, \alpha_b)
$$

(1)

Why would such a function be useful?

2. Find the formula for the curvature $\kappa(0)$ at the first endpoint of a rational Bézier curve segment with first control points $p_0, p_1, p_2$.

3. Show that inflections are invariant under projective transformations.

4. 💀? Given an integral quadratic Bézier curve with control points $p_0, p_1, p_2$, is there an implicit formula $f(p) = 0$ for the curve that is guaranteed to vanish at $p_0$ and $p_2$?

5. Show that a monotonic curve segment with no inflections cannot cross the line that connects its endpoints.

6. Show that the intersection of the tangents at the endpoints of a monotonic curve segment with no inflections happens inside the bounding box.

7. Consider a circle centered at $c$ with radius $r$. Let $f$ be a point in the interior of the circle. Let $p$ be an arbitrary point distinct from $f$. Let $q$ be the intersection between the circle and the ray from $f$ through $p$. Find an expression for the ratio $r = |p - f|/|q - f|$. Now, given a value for $r$, describe the set of points $p$ such that $r = |p - f|/|q - f|$.