CORRIGENDUM TO "A PASTING LEMMA AND SOME APPLICATIONS FOR CONSERVATIVE SYSTEMS"

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As Pedro Teixeira pointed out to us in a personal communication, the statements of some results in Sections 2 and 3 of our paper [1] were incomplete. In fact, we forgot in the statements of Theorems 3.1, 3.2 and 3.3 to add the assumption that the compact set K has "no topology" in the sense that K admits small neighborhoods $K \subset U \subset V$ such that $\Omega = U - V$ is *connected*.

In fact, while this assumption is implicit in our arguments (as it is needed for the application of the results of Dacorogna and Moser), it was not stated explicitly and this might lead to wrong applications of our pasting lemmas.

Indeed, if we *drop* this extra assumption, then the statements of our pasting lemmas are simply *false* as the following counterexample shows. Let $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$ be the usual flat torus and consider $K = ([3/8, 5/8] \times [0, 1])/\mathbb{Z}^2$. Then, it is not hard to see that the conservative vector field $X_0 = \partial/\partial x$ on $(([0, 1] - (1/8, 7, 8)) \times [0, 1])/\mathbb{Z}^2$ can not be glued in a conservative way to the conservative vector field $Y_0 = (1 + \varepsilon)\partial/\partial x$ near K for $\varepsilon \neq 0$.

Of course, this extra assumption (Ω is connected) holds for the applications of the pasting lemmas we proposed in [1] because there the compact set K is a periodic orbit (and thus a compact set without "topology").

Finally, let us point out that the results of Dacorogna-Moser stated as Theorems 2.3 and 3.4 in [1] are not adequate for the proofs of our pasting lemmas because they do not ensure that the vector fields, resp. diffeomorphisms, solving the divergence, resp. determinant, equation glued with the zero vector field, resp. identity diffeomorphism, at the boundary (even though they have the advantage of giving a gain of regularity). In particular, these results should be replaced by their slight variants (without gain of regularity) appearing as Theorem 3 and 4 in [2]. As it turns out, this replacement does not affect the proof of the pasting lemmas for vector fields (i.e., Theorems 3.1, 3.2 and 3.3 in [1]), but it does affect the proof of the pasting lemma for diffeomorphisms, i.e., Theorem 3.6 in [1]. Nevertheless, this statement remains true: indeed, a close inspection of the proof of Theorem 7 in [2] (see e.g. page 15 of [2]) shows that we can glue the derivative $Df(x)$ of a smooth diffeomorphism f at a given point $x \in M$ with f in a conservative way if we place x at the vertex of a cell in the Whitney decomposition constructed in [2].

REFERENCES

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[2] A. Avila, *On the regularization of conservative maps*, Acta Math., v. 205 (2010), 5–18.

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