Do dividends signal more earnings?

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Abstract

Signaling models contributed to the corporate finance literature by formalizing “the informational content of dividends” hypothesis. However, these models are under criticism as the empirical literature found weak evidences supporting a central prediction: the positive relationship between changes in dividends and changes in earnings. We claim that the failure to verify this prediction does not invalidate the signaling approach. The models developed up to now assume or derive utility functions with the single-crossing property. We show that, in the absence of this property, signaling is possible, and changes in dividends and changes in earnings can be positively or negatively related.

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1 Introduction

The information content of dividends is a controversial issue in corporate finance. The research started when Miller and Modigliani (1961) suggested that managers use the dividend policy to convey their expectations of future prospects of the firm. With this hypothesis they proposed to explain the effect of dividend changes on the prices of shares. Since then, theoretical and empirical research advanced. Signaling models were the main tool that formalized the original intuition. Bhattacharya (1979), Miller and Rock (1985), and John and Williams (1985) were the initiators of a long list of signaling models.1 The basic idea is that managers possess private information about the future earnings of the firms and they wish to convey it to the market. However, they cannot simply announce their expectations of future earnings publicly because every firm could imitate them. The information is conveyed by a costly signal. In the cited models, the respective costs are: financing of a committed level of dividends, suboptimal investment and tax on dividends.

On the empirical side, researchers have tried to verify the testable implications derived from the signaling models. In these models, better firms have lower marginal cost of signaling. As a consequence, the models predict that dividends, market price and future or current earnings are positively related. The correlation between dividends and returns is a strongly established result. Aharony and Swary (1980) show that announcements of dividend increase or decrease result in, respectively, positive or negative abnormal returns.

The controversy lays on the relationship between dividends and subsequent earnings. Watts (1973) found positive relation, however, the effect was very small. Healy and Palepu (1988) found a significant relation, but they focused on the particular situation of initiation and omission of dividend payment. Exploring a larger data set Benartzi, Michaely and Thaler (1997) found no significant relation between dividends and future earnings and concluded that dividends are more related to past and present earnings. Recently, Nissim and Ziv (2001), using an improved measure of future earnings, concluded that dividends matter for earnings prediction. Many other papers contributed to this debate and a definitive conclusion seems far to be reached.

We claim that the lack of a clear relation between dividends and earnings is not incom-

1See Allen and Michaely (1995) for a survey on theoretical and empirical issues on dividend policy. See also chapter 6 of Tirole (2005) for asymmetric information models applied to corporate finance.
patible with the information content of dividends. We extend the Miller and Rock (1985) model by including a productivity shock in the production function of the firm. If current earnings and future productivity are positively correlated, dividend payout may represent a higher cost for higher earnings firms because the same sacrifice of investment implies a greater decrease in output for the higher productivity firms. This is the productivity effect. It acts in the opposite direction of the traditional investment effect, in which the cost of dividend is lower for high earnings firms due to the diminishing returns. The interaction between the investment and productivity effects produces a signaling equilibrium in which the level of dividends is a U-shaped function of earnings. For low-earnings firms, the productivity effect dominates and dividends are decreasing in earnings. On the other hand, for high-earnings firms, the investment effect dominates and dividends are increasing in earnings.

This novel U-shaped signaling schedule is a consequence of a peculiarity in the firm’s objective function. Previous models of signaling assume that the objective function of the manager has the single-crossing property, i.e., the marginal cost of signaling is monotonic with respect to the type of the firm. This property is often assumed in signaling models, as in Riley (1979). In dividend models, the single-crossing property generates the monotonic relationship between dividends and earnings. In the model we present in the next section, the objective function does not have the single-crossing property and, consequently, monotonicity is not a necessary characteristic for the signaling function. In the U-shaped equilibrium, two firms that have different earnings decide to signal with the same dividend level, and the market is unable to distinguish between them. This situation is denominated discrete pooling. In the discrete pooling signaling, the dividend payout is an imperfect signal for the quality of the firm and the market evaluates the firm as the average value of the two types of firm.\footnote{To our knowledge, Bernheim (1991) is the only paper that presents a dividend signaling model without the single-crossing property. In that model, the relationship between dividends and quality of the firms is still monotonic.}

\footnote{Discrete pooling is also found in screening problems. See Araujo and Moreira (2001, 2003).}

\footnote{Kumar (1988) presents a dividend signaling model with partial pooling that is similar to our continuous pooling case but generates a monotonic relationship between dividends and quality of the firms.}
We found that, in the U-shaped signaling equilibrium, the market value is still increasing in dividends. But, compared to the traditional separating equilibrium, dividends in the U-shaped schedule are a weaker signal for the future earnings, which can explain why the statistical testing has not succeeded in detecting signaling in dividends. Further, close to the minimum dividend level of the U-shaped schedule, firms that choose the same level of dividends have similar levels of earnings. The difference between the earnings of the firms that choose the same level of dividends is greater for higher dividend levels, so that the variance of the earnings is increasing in dividends. This result suggests that the empirical research testing for information asymmetry should investigate the relationship between signal and variance of the earnings, in order to detect U-shaped signaling.

The model is presented in Section 2. It extends the model of Miller and Rock (1985) by introducing a productivity shock. For concreteness, we assume a quadratic production function in Section 3. Computations of the signaling equilibrium are performed and results are presented in this section. We comment the connections with the empirical literature in Section 4. The conclusions are presented in Section 5.

2 The Model

This model builds on Miller and Rock (1985). There is a firm with production function $F(\cdot)$. The usual properties, $F'(\cdot) > 0$ and $F''(\cdot) < 0$, are assumed. Let $X$ and $Y$ be the earnings, respectively, at period 1 and 2. At the beginning of period 1, managers know $X$ and announce the firm’s financing policy, that is, dividends $d$, and additional funds raised $B$. Then shareholders may sell their shares, dividends are distributed and $X - d + B$ are invested. At the end of period 1, the production of the firm is subject to a multiplicative shock, $\delta > 0$, so that

$$Y = \delta F(X - D),$$

where $D = d - B$ are the net dividends, hereafter simply referred as dividends. At period 2, debts are paid, earnings are distributed and the firm is disassembled. Assume further that $0 \leq D \leq X$. The upper bound for dividends is equivalent to the non-negativity of the investment. As it is realistic that the firms cannot raise funds indefinitely, for tractability, we arbitrarily set the lower bound at zero. The information asymmetry is on the knowledge
of $X$, which is the type of the firm. We assume $X$ is randomly distributed on $[X_1, X_2]$, with density $p(X)$. At period 1, managers know $X$ before the announcement of the dividend, but the market does not, and managers cannot credibly convey their private information to the market. The shock $\delta$ is unknown to both manager and market, but may be correlated with $X$ so that the manager may estimate $\delta$ based on the private information in period 1.

If managers are interested in the maximization of the market value of the firm, they would like to signal $X$ and $\delta$ when the implied value is high. We initially analyze the possibility of signaling of $X$ and $\delta$ separately. The signaling of $X$ is clearly possible. If a firm pays dividends, it incurs in costs due to underinvestment. The concavity of the production function implies that firms that have higher earnings incur in lower marginal cost of dividends. The decreasing marginal cost of dividends allows signaling in which the higher types signal with higher dividends. Also, managers would like to reveal $\delta$ to the market but, in this case, the signaling is not possible. They cannot signal high productivity by distributing more dividends because the sacrifice of output is higher for the high-productivity firms. Nor can they signal by paying fewer dividends because low-productivity firms could imitate them by paying low dividends. However, we will show that if $X$ and $\delta$ are (positively) correlated, the shock in productivity may produce a signaling equilibrium in which, for a subset of firms, the dividend payout is decreasing with respect to the type.

**Assumption 1** Earnings and the multiplicative shock are correlated, and

$$E[\delta|X] = \varepsilon(X) > 0.$$ 

2.1 The value of the firm

At period 1, managers estimate the fundamental cum-dividend value of the firm as the present value of the payment flow,

$$V(X, D) = d + \frac{1}{1+i} \left[ E[\delta F(X - D)|X] - B \right]$$

$$= D + \frac{1}{1+i} \varepsilon(X) F(X - D). \quad (1)$$

Under symmetric information, this is the market value of the shares and managers choose the level of investment in order to maximize $V$. The first-best dividend level, $D^*$,
is given by the Kuhn-Tucker conditions,

\[
F'(X - D^*) - \frac{1 + i}{\varepsilon(X)} \begin{cases}
= 0, & \text{if } D^* > 0, \\
\geq 0, & \text{if } D^* = 0, \\
\leq 0, & \text{if } D^* = X.
\end{cases}
\]  

(2)

As \(D^*\) denotes the net dividends, any change in the gross dividends must be compensated by other funds and Miller and Modigliani (1961) irrelevance result holds.

Under asymmetric information, the market does not know \(X\) and the market value of the shares may not coincide with \(V\). Let \(V^m\) denote the market value. We will assume that \(V^m\) is determined by a signaling equilibrium, that is, firms signal to the market by the choice of the dividend level and the market estimates the value of the firms by observing the dividend choice. The firms will choose dividends above the optimal level, and pay an underinvestment cost for signaling.\(^5\)

Shareholders want to maximize \(V\) if they keep the shares with them until period 2. The ones who intend to sell at period 1 prefer the maximization of the market value, \(V^m\). As in Miller and Rock (1985), we assume the firms’ managers are maximizing a welfare function that aggregates the interests of the shareholders that desire to sell the shares and the ones who do not. Let \(k \in (0,1]\) be the fraction of shareholders that sell at period 1. We assume this fraction is exogenous and it can be motivated by the shareholders’ necessity of liquidity. The welfare function is

\[
W(X, D, V^m) = kV^m + (1 - k)V(X, D).
\]

To analyze the possibility of signaling equilibrium, we are interested in the marginal rate of substitution between \(V^m\) and \(D\). Since \(W\) is quasi-linear with respect to \(V^m\), the relevant properties are found in the marginal welfare of \(D,\)\(^6\)

\[
W_D(X, D) = (1 - k)\left(1 - \frac{1}{1 + i}\varepsilon(X)F'(X - D)\right).
\]

\(^5\)Although signaling equilibrium supported by overinvestment costs is technically possible, it demands relatively high correlation between earnings and the productivity shock. Moreover, the optimal investment level must be sufficiently low. We restrict our analysis to the underinvestment case which is more plausible when the resources for investment are scarce.

\(^6\)We use subscripts to denote partial derivative with respect to the subscripted variable.
For a given level of dividends, an increase in $X$ reduces $F'$, but may increase $\varepsilon$. Thus, a positive relationship between the productivity shock and earnings may make $W_D$ to be non-monotone with respect to the type. Or, equivalently, the cross-derivative of $W$,

$$W_{XD}(X, D) = \frac{1 - k}{1 + \frac{1}{d}} \left[ -\varepsilon(X)F''(X - D) - \varepsilon'(X)F'(X - D) \right], \quad (3)$$

may change its sign. We can define two regions in the $(X, D)$ space, according to the sign of $W_{XD}$.

**Definition 1** The $CS^+$ region (resp. $CS^-$ region) is the set of points in the $(X, D)$ space such that $W_{XD}(X, D) > 0$ (resp. $W_{XD}(X, D) < 0$).

In (3), the first term in the brackets is the investment effect. Firms with higher earnings invest more and have lower marginal product. Consequently, the marginal cost of dividend is lower for higher types. The second term is the productivity effect. Earnings provide information about the future productivity, which affects the expected marginal cost of the signal.

**Negative correlation between earnings and productivity**

When $\varepsilon'(X) \leq 0$, higher earnings reduce the expected productivity and the cost of signaling is lower. The welfare function has the single-crossing property since both productivity and investment effects collaborate on $W_{XD} > 0$. The results are, therefore, similar to the ones found by Miller and Rock (1985). Dividends and earnings are positively related.

**Positive correlation between earnings and productivity**

If $\varepsilon'(X) > 0$, higher earnings correspond to higher optimal investment levels and dividends become costlier since more earnings should be retained for investment. If, for some combination of type and dividend level, the productivity effect dominates, then $W_{XD} < 0$ and higher types will be more reluctant to pay dividends because of the loss of investment opportunities. Conversely, $W_{XD} > 0$ holds when the investment effect dominates. In this case, lower types are more reluctant to pay dividends because they have fewer investment resources. Note that from (2) the first-best dividend level as a function of types, $D^*(X)$, is increasing for $W_{XD} > 0$ and decreasing for $W_{XD} < 0$. If the investment effect dominates,
firms with higher earnings may pay more dividends. If the productivity effect dominates, they should invest more by paying lower dividends. When \( \varepsilon'(X) > 0 \), the sign of \( W_{XD} \) is ambiguous and the single-crossing property does not hold.\(^7\) The next step is to characterize the signaling equilibrium in this context. We show below that the relationship between earnings and dividends may be negative for lower types and positive for higher types.\(^8\)

### 2.2 The signaling equilibrium

Signaling is defined as a perfect Bayesian equilibrium (the formal definition is provided in Appendix A.1). The basic description is the same for both the single-crossing and the non-single-crossing case. The market generates a value function, \( V^m(\cdot) \), which assigns a market value \( V^m(D) \) to any firm that announces the dividend level \( D \). Each firm takes \( V^m(\cdot) \) as given and chooses the dividend level that maximizes \( W \). We have a signaling equilibrium if the zero expected profit condition holds, that is,

\[
V^m(D) = E_\mu[V(X, D)|D],
\]

where \( E_\mu \) denotes the expectation taken on the Bayesian updated distribution on \( X \). The market value \( V^m \) is the expected value of the firm with respect to the probability distribution of \( X \) that results from the Bayesian update given the choice of \( D \) by the firm.

Formally, solving the signaling problem consists in finding functions \( V^m(X) \) and \( D(X) \) such that the type \( X \) firm chooses a dividend level \( D(X) \) and is evaluated as \( V^m(X) \) by the market. Since dividends and market value are linked by \( V^m(\cdot) \), these functions are related by \( V^m(X) = V^m(D(X)) \).

Define the welfare of a type \( X \) firm that declares to be type \( \hat{X} \) as

\[
W(\hat{X}, X) = W(X, D(\hat{X}), V^m(D(\hat{X})))
= kV^m(D(\hat{X})) + (1 - k)V(X, D(\hat{X})).
\]

\(^7\)An alternative setting, without the multiplicative shock, that generates an non-single-crossing objective function would be to assume that the production function has increasing returns for low levels of investment and decreasing returns otherwise. In this case, \( W_{XD} = \frac{-k}{X+1}F''(X-D) \) and, consequently, marginal cost of dividends increases with earnings only for firms in increasing returns.

\(^8\)Araujo and Moreira (2003) develop a competitive screening model applied to insurance market that is similar to a signaling model.
In order to be incentive compatible, each firm should prefer to tell the truth, that is,

\[ W(X, X) \geq W(\hat{X}, X), \tag{5} \]

for all \( X, \hat{X} \in [X_1, X_2] \). The corresponding first order condition,

\[ \frac{\partial W}{\partial X}(X, X) = 0, \tag{6} \]

provides a differential equation that characterizes candidate dividend schedules, \( \mathcal{D}(X) \). Additionally, the second order condition constrains the slope of \( \mathcal{D}(X) \), as summarized in the following proposition.

**Proposition 1** In a signaling equilibrium, \( \mathcal{D}(X) \) is non-decreasing in the \( CS^+ \) region and non-increasing in the \( CS^- \) region.

*Proof:* See the Appendix.

When the single-crossing property is present, \( W_{XD}(X, D) \) has the same sign everywhere. Therefore, \( CS^+ \) and \( CS^- \) do not show up simultaneously and, by Proposition 1, contracts should be monotone. As a consequence, types are separated when \( \mathcal{D}'(X) \neq 0 \), or a interval of types is bunched when \( \mathcal{D}'(X) = 0 \). When the single-crossing property does not hold, monotonicity is not assured. The relationship between type and signal may present a variety of shapes, that combines separation, pooling and U-shaped segments. When signaling is U-shaped, firms in a disconnected set of types signal with the same dividend level.

Eq. (6) and Proposition 1 are local conditions for incentive compatibility. In the single-crossing case, these conditions are sufficient for incentive compatibility. However, when single-crossing does not hold, as in the model presented here, signaling schedules that satisfy the conditions may fail to be globally incentive compatible. As there are no analytical conditions for global incentive compatibility, it must be verified by numerical methods.

The following proposition establishes the positive relationship between market value and dividends, even when the single-crossing property does not hold.

**Proposition 2** If a firm pays dividends above the first-best dividend level, then market value and dividends are positively related.

*Proof:* See the Appendix.
2.3 Equilibria diversity

In an equilibrium, the same signal, $D$, may be chosen by many types. We are interested in classifying the equilibrium according to its degree of separability. The following definition will be useful:

**Definition 2** The pooling set, $\Theta(D)$, is the set of types whose signal is $D$, that is, $\Theta(D) = \{X \in [X_1, X_2] | D(X) = D\}$.

In particular, in a separating equilibrium, $\Theta(D)$ is singleton for every $D$ that is chosen by a firm.

**Definition 3** The type $X$ is separated if $\Theta(D(X)) = \{X\}$. A separating equilibrium is a signaling equilibrium such that every $X$ is separated.

When $X$ is separated, the market correctly infers the type by the observation of $D$. So $V^m(D) = V(X, D)$, where $X$ is the type that chooses the dividend level $D$.

**Proposition 3** In an interval of separated types, $D(X)$ follows the differential equation

$$D'(X) = \frac{-kV_X(X, D(X))}{V_D(X, D(X))}.$$ (7)

*Proof:* See the Appendix.

As in the single-crossing case, a pooling equilibrium may be defined as a continuum of types that choose the same signal level.

**Definition 4** The type $X$ is continuously pooled, if there is a non-degenerate closed interval $I$, such that $X \in I \subset \Theta(D(X))$. A continuous pooling equilibrium is a signaling equilibrium such that, for every $X$, $\Theta(D(X)) = [X_1, X_2]$.

In signaling games without the single-crossing condition, a new kind of pooling arises. As in the continuous pooling equilibrium, some values of $D$ will be chosen by more than one type of firm. However, the number of pooled types may be finite.

**Definition 5** The type $X$ is discretely pooled, if $X$ is an isolated point of $\Theta(D(X))$ and $\Theta(D(X)) \neq X$. 
The property aggregating the discretely pooled types is that they have the same marginal value $V_D$.

**Proposition 4** If $X_a$ and $X_b$ are discretely pooled, $D = \mathcal{D}(X_a) = \mathcal{D}(X_b)$, $\mathcal{D}'(X_a) \neq 0$, and $\mathcal{D}'(X_b) \neq 0$, then

$$V_D(X_a, D) = V_D(X_b, D).$$

(Eq. 8)

**Proof:** See the Appendix.

Eq. (8) gives $\varepsilon(X_a)F'(X_a - D) = \varepsilon(X_b)F'(X_b - D)$. So different types can choose the same level of dividend when, for higher types, the higher productivity shock compensates the reduction in marginal productivity resulted from the higher investment. In the discrete pooling, the dividend choice does not fully reveal the type of the firm. The market knows the set of possible types but it cannot distinguish one type from the other. This fact is taken into account when the market estimates the value, so $E_\mu[V(X,D)|D]$ is the average value of the types in the pool.

**Assumption 2** The type $X$ is uniformly distributed on the interval $[X_1, X_2]$.

Under Assumption 2, each type has the same probability. In particular, when there are only two types in the pool, the expected value of the firm is

$$E_\mu[V(X,D)|D] = \frac{1}{2}V(X_a, D) + \frac{1}{2}V(X_b, D),$$

where $X_a$ and $X_b$ are the types that choose $D$.

**Proposition 5** Under Assumption 2, in a interval with discretely pooled types, if exactly two types choose the same dividend, $\mathcal{D}(X)$ follows the differential equation

$$\mathcal{D}' = \frac{-k[V_X(X, D) + V_X(X(X, D), D)\overline{X}_X(X, D)]}{kV_X(X(X, D), D)\overline{X}_D(X, D) + 2V_D(X, D)},$$

(Eq. 9)

where $\overline{X}(X, D)$, derived from (8), is the type pooled together with type $X$, when dividend $D$ is chosen.

**Proof:** See the Appendix.
2.4 Equilibrium refinement

The disturbing fact in signaling models is that the equilibrium is not unique. For the same parameters, different kinds of equilibrium may exist, and the choice of initial conditions may generate a continuum of equilibria. At this point a selection criterion is needed. When the single-crossing property holds and, consequently, the relationship between type and signal is monotone, the natural choice is the equilibrium in which the lowest type chooses the first-best level of signal. This is the most efficient equilibrium among the separating equilibria. Such a criterion is not suitable for non single-crossing models, as monotonicity may not be valid. The pro-separation criterion, defined below, chooses the one that minimizes pooling and maximizes efficiency.

Assumption 3 Separability degree of a continuous pooled type, a discretely pooled type, and a separated type are, respectively, 1, 2, and 3.

Definition 6 Let \( \Pi(d) = \{X \in [X_1, X_2] | X \text{ has separability degree } d\} \).

Therefore \( \Pi(1) \) is the set of continuously pooled types, \( \Pi(2) \) is the set of discretely pooled types and \( \Pi(3) \) is the set of separated types.

Definition 7 The separation floor of a signaling equilibrium is the lowest separability degree associated to a type in \([X_1, X_2]\).

Definition 8 A pro-separation equilibrium is a signaling equilibrium with separating floor \( \varphi \), such that (a) there is no other equilibrium with higher separation floor; (b) among equilibria with same separation floor, there is no other with lower probability of \( \Pi(\varphi) \), according to density \( p(\cdot) \); and (c) among equilibria with same separation floor and same probability of \( \Pi(\varphi) \), there is no other equilibrium with higher expected value, according to density \( p(\cdot) \).

Therefore, the pro-separation equilibrium criterion chooses an equilibrium eliminating poorly separated equilibria and taking the most efficient among the surviving equilibria.
3 The Quadratic Case

The complexities of this model, introduced by the global incentive compatibility and the pro-separation criterion, require a numerical approach for the characterization of the equilibrium. For the computations we consider a quadratic production function,

$$F(I) = aI(b - I),$$

where $0 \leq I \leq b/2$, $a > 0$, and $b > 0$. We assume a linear expected productivity shock

$$\varepsilon(X) = g + hX,$$

where $h > 0$.

By (3), we have the cross derivative of the welfare function,

$$W_{XD} = \frac{4ah(1-k)}{1+i} \left( X - \frac{D - b}{2} + \frac{g}{2h} \right).$$

The frontier between the $CS+$ and $CS-$ regions is a positively sloped straight line described by

$$X = \frac{D}{2} + \frac{b}{4} - \frac{g}{2h},$$

and the $CS+$ region is observed for $X$ above or $D$ below the frontier. Only two types can be present in the discrete pooling equilibrium, and these types are mutually associated by the function $X$, derived from the marginal condition (8). Thus,

$$X(X,D) = -\frac{g}{h} + \frac{b}{2} + D - X.$$

For a given level of dividend, the firm with the higher type is in the $CS+$ region and pools with the one with the lower type in the $CS-$ region. The marginal cost of signaling is decreasing with respect to the types, for high types, due to the investment effect, and is increasing for low types, due to the productivity effect.

The differential equation for the separating equilibrium is derived from (7),

$$\mathcal{D}' = \frac{-ka \left[ -3hX^2 + 2(2hD + hb - g)X - hD^2 + (2g - hb)D + bg \right]}{-2a(hX + g)D + a(2hX^2 + (2g - hb)X - bg) + 1 + i},$$

and, from (9), the differential equation for the discrete pooling equilibrium is

$$\mathcal{D}' = \frac{-k \left( \mathcal{D} + \frac{b}{2} + \frac{g}{h} \right) (2X - (\mathcal{D} + \frac{b}{2} - \frac{g}{h}))}{R(X, \mathcal{D})},$$

(14)
where
\[ R(X, D) = (4 - 3k)X^2 - \left[(2 - k)(b + 2D) - 4(1 - k)\frac{g}{h}\right]X \]
\[- (2 - k)(b + 2D)\frac{g}{h} + k \left(\frac{b^2}{4} - \frac{g^2}{h^2}\right) + \frac{2(1 + i)}{ah}.\]

The next proposition shows that firms underinvest in discrete pooling signaling.

**Proposition 6** In the quadratic specification, discretely pooled types signal with dividend level above the first-best.

**Proof:** See the Appendix.

Equipped with the differential equations (13) and (14), numerical routines compute the pro-separation equilibrium taking into account the incentive compatibility condition and pro-separation criterion.

### 3.1 An example

As a first example, we choose parameter values that generate a representative equilibrium with discretely pooled and separated types. In this example, we assume \( a = 1, b = 4, \ i = 0, k = \frac{1}{2}, \ g = 0, \ h = 1, \) and \( X \) uniformly distributed over \([X_1, X_2] = [0.8, 2]\).

In this context we have \( W_{XD} = 2X - D - 2, \) and the \( CS+ \) region is defined by \( X > \frac{D}{2} + 1. \) The thick line in Figure 1 splits the \((X, D)\) space in the \( CS+ \) and \( CS- \) regions. Signaling schedules, \( D(X), \) cannot lay on the region above \( D = X \) because investment in period 1 is not negative. By Proposition 1, \( D(X) \) is non-increasing in the \( CS- \) region and non-decreasing in the \( CS+ \) region.

The grayed region in Figure 1 is the region where discrete pooling is possible for our choice of support. For each level of dividend, every type \( X \) in the grayed region may be discretely pooled with the corresponding type \( \overline{X}(X, D) = D + 2 - X, \) which is inside the grayed region. If \( X \) is in the \( CS+ \) region, then \( \overline{X}(X, D) \) is in the \( CS- \) region, and if \( X \) is in the \( CS- \) region, then \( \overline{X}(X, D) \) is in the \( CS+ \) region. For low dividends and high types, there are points in the \( CS+ \) region that are located outside the grayed region. These points cannot be discretely pooled because \( \overline{X}(X, D) < X_1, \) that is, the candidates for pooling is out of the support of types.
From (13), separated types signal according to the differential equation

$$D' = \frac{3X^2 - 4(D + 2)X + D(D + 4)}{2 - 4X(2 - X + D)},$$

(15)

and, from (14), the discrete pooled types signal according to

$$D' = \frac{-2(D + 2)X + (D + 2)^2}{5X^2 - 6(D + 2)X + 8}.$$  

(16)

First we investigate the possibility of separating equilibrium. By analyzing the sign of (15), we conclude that $D'(X) > 0$, for $X \in [0.8, 1]$ and $0 \leq D \leq X$. As dividends for $X \in [0.8, 1]$ are in the $CS-$ region, Proposition 1 states that the types in this interval may not be separated. Thus, there should be some kind of pooling in any signaling equilibrium.

Signaling with discretely pooled types follows (16). Some solutions are shown in Figure 2, and the thick curve is the V-shaped solution of (16). The solutions for which the local incentive compatibility condition of Proposition 1 holds, that is, solutions that are non-increasing in the $CS-$ region and non-decreasing in $CS+$ region. The V-shaped solution and all the U-shaped solutions above it satisfy the condition of Proposition 1. Any other solution does not provide an incentive compatible signaling for discretely pooled types. The dashed curve is the first-best dividend level $D^*$ given by (2). For $X \in [1 - \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}]$ the firm should not pay dividends and invest all the earnings. For all other types, $D^* = X - 2 + \frac{1}{2X}$. The V-shaped solution is the most efficient discretely pooled signaling in the sense that it is the closest solution to the first-best dividend level, for every type.

The locally incentive compatible solutions of (16) may not represent a complete description of the signaling equilibrium. For instance, the V-shaped solution is not fully contained in the grayed region of discrete pooling. As it crosses the boundary of the grayed region at $X_s$, the types in $(X_s, X_2]$ may not be discretely pooled and their signal does not follow the V-shaped solution. These types may be separated by a schedule that follows the differential equation (15). Figure 3 shows a signaling equilibrium $D(X)$ that coincides with the V-shaped solution of (16) in $[X_1, X_s]$, and is a solution of (15) in $(X_s, X_2]$. In this way, types are discretely pooled in $[X_1, X_s]$ and are separated in $(X_s, X_2]$.

---

9The V-shaped solution is a saddle solution of (16). Its vertex is the singularity point where $D'(X)$ is not defined, and it is the limit point of the right side and left side solutions.
The choice of the separating schedule in \((X_s, X_2)\) among the solutions of (15) is not casual. The differential equation is solved after the initial point \((X_s, D_s)\) is chosen, where 
\[
D_s = \lim_{X \to X_s^+} D(X).
\]
And \(D_s\) is the root of
\[
W(X_s, D(X_s), V^m(X_s)) = W(X_s, D_s, V(X_s, D_s)),
\]
which states that the type \(X_s\) is indifferent between the signaling in the discrete pooling or signaling in the separating schedule. Figure 4 shows the market value function \(V^m(D)\) corresponding to the signaling schedule in Figure 3. According to (17) the type \(X_s\) is indifferent between points \(P\) and \(Q\). This condition preserves the incentive compatibility in the neighborhood of \(X_s\). Otherwise types in the right neighborhood of \(X_s\) may prefer to signal in the discrete pooling schedule or types in the left neighborhood of \(X_s\) may prefer to signal in the separation schedule.

In the discrete pooling equilibrium, lower types pool with higher types because, in \(CS\), benefits from being treated equally as higher types compensate the cost of higher dividend distribution. The market knows the disguising behavior and adjust the evaluation, but signaling is preserved since lower types do not reduce the expect valuation to the point that higher types refrain from signaling.

There is a discontinuity in \(D(X)\) and \(V^m(X)\) at \(X_s\) where signaling changes form discrete pooling to separation. For \(D = D(X_s)\), the market value is an equally weighed average of \(V(X_1, D(X_1))\) and \(V(X_s, D(X_s))\). If \(D(X_s)\) were continuous, then for a small \(\eta > 0\) the signal of \(X_s + \eta\) would be near \(D(X_s)\). But the market correctly infers that \(X_s + \eta\) is separated so the market value associated to the signal should be near \(V(X_s, D(X_s))\). As \(V(X_1, D(X_1)) < V(X_s, D(X_s))\), there must be a upward jump in \(V^m(D)\) and it could not be incentive compatible because \(X_s\) would prefer to signal as \(X_s + \eta\) to obtain a discontinuously greater market value. The discontinuity in the signaling is a consequence of Assumption 2. If the probability density were continuous and equal to zero at \(X_1\), the discontinuity could be eliminated.

Whenever the single-crossing property holds, the first and second order conditions of the firm’s maximization problem are sufficient for global maximum. Then, the differential equations derived from these conditions generate incentive compatible signaling schedules, provided that the conditions in Proposition 1 are satisfied. However, without the single-crossing condition, global incentive compatibility is not assured. In the quadratic case,
sometimes a discretely pooled type prefers a signal assigned to a separated type. In this case, this signaling schedule is not a valid signaling equilibrium. So, an additional checking for incentive compatibility is needed. This checking is performed computationally and we found that the equilibrium of our example, illustrated in Figure 3, is globally incentive compatible.

The signaling equilibrium we found for the example was based on the choice of the V-shaped solution of discretely pooled types. However, we could choose a U-shaped solution above it V-shaped solution and find another signaling equilibrium combining an appropriate separated schedule. Further, we show below that discrete pooling combined with separation is not the only kind of signaling equilibrium. The pro-separation equilibrium is our criterion for equilibrium selection. In the example, the equilibrium based on the V-shaped solution is the pro-separation equilibrium because the signaling equilibria based on U-shaped solutions has a shorter interval of separated types.

3.2 Other cases

The combination of discrete pooling with separation is not the only kind of signaling equilibrium. Depending on the support of distribution and the parameter values, other kinds of equilibrium may arise. Figure 5 shows other three cases. In graph (a) the CS-set is small and a separating schedule for all types fits in CS+ region. In graphs (b) and (c) the V-shaped solution crosses the $D = X$ line and consequently lower types cannot signal in a discrete pooling. In this case some types may be in continuous pooling (bunching). Graph (b) shows a discrete pooling with bunching. A continuum of lower types pools with a continuum of higher types so that the value of the pool coincides with the limit value of the discrete pools, for the same level of dividend. In graph (c), the V-shaped solution requires a higher level of dividends, such that discrete pooling is not possible even if combined with bunching. The equilibrium is a continuous pooling of all types at the efficient dividend level. Graph (d) shows the three cases on the $(D, V^m)$ space.

3.3 Changing the parameter values

Figure 6 shows equilibria for some values of the parameter $k$, maintaining the values of the remaining parameter equal to the first example. The parameter $k$ represents the weight of
market price in the objective of the manager. For higher $k$, the manager is more inclined to sacrifice future earnings, by paying dividends in exchange for an increase in $V^m$, or, equivalently, a small increase in $V^m$ is sufficient to induce the manager to pay a given level of dividends. Consequently, indifference curves and the signaling equilibrium at the $(D, V^m)$ space is flatter when $k$ is higher. Comparing the equilibrium for $k = 0.5$ and $k = 0.7$, it is noticeable that the V-shaped solution for the higher $k$ is at a higher dividend level. The flatter is the indifference curve, the higher must be the dividend in order to obtain local incentive compatibility, and, as the lowest types cannot pay the dividends required for discrete pooling, the equilibrium with bunching occurs. On the other side, for $k = 0.1$ and $k = 0.3$ the V-shaped solution is not an equilibrium because the global incentive compatibility condition does not hold. For lower $k$, the relatively high level of $V^m$ increases the probability of violation of the global incentive compatibility condition. Consequently, the signaling equilibrium is higher and the set of separated types is shorter.

The change of the parameter $g$ is examined in Figures 7 and 8. We recalculate the equilibria for $g = -0.2$ and $g = 0.2$. For lower $g$, the relative role of the productivity shock on the value of the firm is greater, and, as lower types have more opportunity to pool with higher types, pooling is prevalent in Figure 7. For higher $g$, the importance of the productivity shock is diminished and separation is more likely as is shown in Figure 8. For parameter $h$, the rationale is similar. The greater is the relative role of the productivity shock, the stronger is the productivity effect, and pooling is more likely. Conversely, for weaker productivity effect, separation is favored as the model is closer to Miller and Rock (1985).

Geometrically speaking, the influence of the parameters $g$, $h$ and $b$ on the equilibrium is determined by the placement of the $CS+/CS-$ frontier. By (12), higher $b$, lower $g$, higher $h$ for positive $g$, or lower $h$ for negative $g$ moves the frontier to right and the $CS-$ region enlarges. Consequently, separating equilibria are less frequent and continuous pooling more common. On the other side, changing the value of the parameters in the opposite direction, the $CS+$ region enlarges and separating equilibria are more frequent. Discrete pooling is an intermediate case that will be present when the $CS+$ and $CS-$ regions coexist in a favorable way.
4 Connections to the Empirical Literature

4.1 Future earnings

In the equilibria computed in Section 3, we analyzed the relationship between dividends, \( D \), and current earnings, \( X \). As empirical studies are concerned about the future earnings, \( Y \), we plot in Figure 9 the relationship between dividends and expected future earnings, \( \mathbb{E}[Y] \), corresponding to the first example developed in Section 3. The essential point is that the segment of the curve associated to the discretely pooled types is flatter compared to the separated type segment. This means that discrete pooling weakens the ability of dividends to signal. For the low dividend firms, large changes in dividends are related to relatively small changes in future earnings. On the other hand, high dividends firm are in the subset of separated types and maintain the ability to signal.

4.2 Testable implications

From the equilibria we found, two testable implications arise. First, considering the set of discretely pooled types that signal with a given level of dividends, the variance of earnings is increasing with respect to dividends. For low earnings firms in discrete pooling, the productivity effect dominates. As earnings and productivity shocks are positively correlated, higher earning firms expect higher productivity in future and are more reluctant to sacrifice investment. Consequently, dividends are decreasing with respect to earnings. On the other hand, if earnings are high enough, decreasing returns of investments dominates over productivity shock, and the original Miller and Rock (1985) signaling reappear, that is, higher earnings firms pay more dividends because the signaling cost is lower for them. The resulting signaling schedule is U-shaped, and high dividends are used as a signal by more distant types. Conversely, firms that signal with low dividend have similar earnings. This result suggests that the variance of earnings is increasing in dividends, and such relationship can be used as a test for our model when the single-crossing property does not hold. Moreover, as earnings are ex post observable, the U-shaped curve may be directly identified by a regression that includes the square of earnings, \( X^2 \).

The second implication is that, when separation and discrete pooling coexist, there may be a jump in the signaling schedule, as illustrated in Figure 3. As low earnings firms
are in discrete pooling set, the average earning and, consequently, the dividend level is substantially lower in this class. A statistical analysis may detect a bimodal distribution of the signal, such that higher levels correspond to the separated types and lower levels, to the discretely pooled types. Furthermore, for discretely pooled types the signaling is weaker, that is, the corresponding earnings must be confined to a smaller interval.

4.3 Other remarks

The correspondence between the empirical results and the implications from signaling models is imprecise. The main point is that the signaling approach characterizes single-period dividends. Since the empirical analysis, as in Benartzi, Michaely and Thaler (1997), focus on changes in dividends and changes in earnings, the correspondence should be made to variables of a dynamic model, which is out of the scope of this work. The empirical literature implicitly assumes that the signaling profile is stable, thus, changes in dividends and changes in earnings are positively related in separating equilibrium.

When signal is non-monotonic, the same conclusion does not hold. The relationship between changes in dividends and the average change in earnings depends on the distribution of the changes, about which we did not make any assumption. As the signal decreases with respect to earnings for some types, and increases for others, the resulting relationship is ambiguous. The equilibrium we found suggests that the empirical research should also examine the relationship among the levels of the variables.

5 Conclusion

Traditional models of signaling satisfy the single-crossing property for the objective function of the firm. This property leads to a monotone relationship between types and signals. We believe that this assumption is not essential and the results derived from non-single-crossing signaling are also plausible. For instance, in our dividend-signaling model, a low-earning firm may pay high dividends in order to be considered a high-earning firm. The general result is that the monotone relationship between types (current earnings) and signals (dividends) is not assured and the signaling power of dividends is weakened.

This result conveys a message to the empirical research that tests for the presence of
asymmetric information assuming the single-crossing property. Critics on the “informational content of dividends” hypothesis claim that the statistical evidences of positive relation between dividends and earnings are weak. But, as we have shown, this is a possible result when the discrete pooling equilibrium occurs. Our results suggest that future empirical researches should identify separated and pooled types, and investigate the relation between variance of earnings and dividends. Or, as earnings are ex post observable, a direct identification of the U-shaped dividend schedules may be attempted.

We believe that the absence of the single-crossing property is not an uncommon situation and the empirical research must take this possibility into account. The presence of discrete pooling enriches the interrelation among variables in signaling models and tests based simply on monotonicity should not be used to reject asymmetric information.

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A Appendix

A.1 The perfect Bayesian equilibrium

A perfect Bayesian equilibrium (PBE) for the dividend signaling model is a profile of strategies \( \{e(X) = (D(X), Y^m(X)) \}_{X \in [X_1, X_2]} \) and ex-post beliefs \( \mu(\cdot | c) \) such that the following conditions are satisfied:

\(^{10}\)For instance, see Araujo and Moreira (2003) for the insurance case.
1. Zero expected profit constraint:

\[ \gamma^m(X) = \int V(\hat{X}, D(X))d\mu(\hat{X}|c(X)). \]

2. Maximization of the firm welfare:

\[ X \in \arg \max_{\hat{X} \in [x_1, x_2]} k\gamma^m(\hat{X}) + (1 - k)V(X, D(\hat{X})). \]

3. Consistency of beliefs: \( \mu(X|c) \) is the Bayesian updating given 1 and 2, i.e., it is the probability a posteriori of \( X \) given \( c \).

### A.2 Proof of propositions

#### A.2.1 Proof of Proposition 1

The first order condition (6) can be written as

\[ W_D(X, D(X))D'(X) + kV''(D(X))D'(X) = 0, \]

whose derivative is

\[ W_{XD}(X, D(X))D'(X) + \left[ W_{DD}(X, D(X)) + kV'''(D(X)) \right] (D'(X))^2 \]
\[ + \left[ W_D(X, D(X)) + kV''(D(X)) \right] D''(X) = 0. \]  

(18)

From the definition of \( W \),

\[ \frac{\partial^2 W}{\partial X^2}(\hat{X}, X) = \left[ W_{DD}(X, D(\hat{X})) + kV'''(D(\hat{X})) \right] (D'(\hat{X}))^2 \]
\[ + \left[ W_D(X, D(\hat{X})) + kV''(D(\hat{X})) \right] D''(\hat{X}). \]  

(19)

Equations (18) and (19) simplify the second order condition to

\[ \frac{\partial^2 W}{\partial X^2}(X, X) = -W_{XD}(X, D(X))D'(X) \leq 0, \]

which proves the proposition.
A.2.2 Proof of Proposition 2

From the first order condition (6),

\[
\left[ k \frac{dV}{dD}(D(X)) + (1 - k)V_D(X, D(X)) \right] D'(X) = 0.
\]

The first-best dividend is given by \( V_D(X, D) = 0 \). As \( V_D(X, D) \) is a decreasing in \( D \), \( V_D < 0 \) occurs above the first-best level. Consequently, from (20), for \( D'(X) \neq 0 \),

\[
\frac{dV}{dD} > 0.
\]

A.2.3 Proof of Proposition 3

Since the type is perfectly revealed by the observation of \( D \), the market correctly evaluate the firm, that is, \( V^m(D) = V(X, D) \), where \( X = D^{-1}(D) \). The welfare is

\[
W(\hat{X}, X) = kV(\hat{X}, D(\hat{X})) + (1 - k)V(X, D(\hat{X})),
\]

and the first order condition (6) implies (7).

A.2.4 Proof of Proposition 4

The first order condition is

\[
k \frac{dV}{dD}(D(X))D'(X) + (1 - k)V_D(X, D(X))D'(X) = 0,
\]

and, since \( D'(X) \neq 0 \),

\[
V_D(X, D(X)) = -\frac{k}{1 - k} \frac{dV}{dD}(D(X)).
\]

As \( D(X_a) = D(X_b) \), the right hand side is constant for types in the same pool. Therefore,

\[
V_D(X_a, D(X_a)) = V_D(X_b, D(X_b)).
\]

A.2.5 Proof of Proposition 5

Let \( X \) be the lowest type that chooses \( D \) and \( \overline{X}(X, D) \) the highest. The market value is an average value of the pooled types in the same dividend level,

\[
V^m(D(X)) = \frac{1}{2} V(X, D(X)) + \frac{1}{2} V(\overline{X}(X, D(X)), D(X)),
\]
and the welfare as a function of the firm’s declared and true types is

\[ W(\hat{X}, X) = \frac{k}{2} V(\hat{X}, D(\hat{X})) + \frac{k}{2} V(X, D(\hat{X})) \]

Differentiating in \( \hat{X} \),

\[
\frac{\partial W}{\partial \hat{X}}(\hat{X}, X) = \frac{k}{2} \left[ V_X(\hat{X}, D(\hat{X})) + V_D(\hat{X}, D(\hat{X})) \partial_D'(\hat{X}) \right] \\
+ \frac{k}{2} \left[ V_X(X, D(\hat{X})), D(\hat{X})) \left( \sum_X(X, D(\hat{X})) + \sum_D(\hat{X}, D(\hat{X})) \partial_D'(\hat{X}) \right) \right] \\
+ \frac{k}{2} \left[ V_D(X, D(\hat{X})), D(\hat{X}) \partial_D'(\hat{X}) \right] \\
+ (1 - k) V_D(X, D(\hat{X})) \partial_D'(\hat{X}),
\]

and taking into account that (8) implies \( V_D(X, D(X)) = V_D(\sum_X(X, D(X)), D(X)) \), and that truth-telling implies \( \hat{X} = X \), condition (6) gives

\[
\frac{k}{2} V_X(X, D(X)), D(X)) \left( \sum_X(X, D(X)) + \sum_D(X, D(X)) \partial_D'(X) \right) \\
+ \frac{k}{2} V_X(X, D(X)) + V_D(X, D(X)) \partial_D'(X) = 0.
\]

And, solving for \( \partial_D'(X) \), we establish (9).

**A.2.6 Proof of Proposition 6**

Differentiating \( V(X, D) \) with respect to \( X \),

\[
V_X(X, D) = \frac{1}{1+i} \left[ \varepsilon'(X) F'(X - D) + \varepsilon(X) F'(X - D) \right].
\]

As \( \varepsilon'(X) = h > 0 \), then \( V_X(X, D) > 0 \). Moreover, \( \sum_X(X, D) = -1 \) and \( \sum_D(X, D) = 1 \). And, taking into account that \( X \) and \( \sum(X, D) \) are discretely pooled, the numerator of (9) is

\[
-k \left[ V_X(X, D) + V_X(\sum(X, D), D) \sum_X(X, D) \right] = -\frac{hk}{1+i} \left[ F(X - D) - F(\sum(X, D) - D) \right] \\
= -\frac{ahk}{1+i} \left( D + \frac{b}{2} + \frac{g}{h} \right) \left( 2X - \left( D + \frac{b}{2} - \frac{g}{h} \right) \right) \\
= -\frac{k}{2(1-k)} \left( D + \frac{b}{2} + \frac{g}{h} \right) W_{XD}(X, D).
\]

By the constraint \( I < b/2, X < D + b/2 \), and, as \( \varepsilon(X) > 0, X > -g/h \). Therefore,

\[
D + \frac{b}{2} + \frac{g}{h} > 0,
\]
and the sign of the numerator of (9) is the opposite of $W_{XD}(X, D)$. By Proposition 1, $D'(X)$ must have the same sign of $W_{XD}(X, D)$. Therefore, the denominator of (9) is negative. As $V_X(X, D) > 0$ and $X_D(X, D) = 1$, we conclude that $V_D(X, D) < 0$, that is, dividends are above the first-best level.

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Figure 1: CS split and discrete pooling region.

Figure 2: Discrete pooling paths.
Figure 3: Signaling equilibrium.

Figure 4: Market value and dividends.
Figure 5: Signaling equilibria.

Figure 6: Equilibria for different values for $k$. 
Figure 7: Equilibria for $g = -0.2$.

Figure 8: Equilibria for $g = 0.2$. 
Figure 9: Expected future earnings.