

HOMOLOGIC AND HOMOTOPIC INVARIANTS OF GROUPS

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In this mini-course homological and homotopical properties of groups will be considered. We will study the homological and homotopical invariants of (discrete, finitely generated) groups suggested by R. Bieri, W. Newmann, R. Strebel, Renz. These invariants (often called geometric) are linked with the homological types FP_m and the homotopical types F_m of groups. The geometric invariants classify when a subgroup H of a group G containing the comutator $[G, G]$, where G has type FP_m (respectively F_m), has the same homological type FP_m (respectively the same homotopical type F_m). One interesting corolary of the theory of the geometric invariants of groups is a result of R. Bieri and R. Strebel that for a finitely presented group G either G has a free subgroup of rank 2 or every metabelian quotient of G is finitely presented (= has homotopical type F_2). By now the geometric invariants have been calculated in few cases and even the case of metabelian group G is still open, though in this case there are 2 outstanding conjectures : the FP_m -Conjecture and the Σ^m -Conjecture. We will outline the current stage of the conjectures. If the time permites we will consider some recent results on related problems for pro- p groups and Lie algebras. The methods used in the theory of the geometric invariants usually have both algebraic and geometric flavour. We will emphasize on the algebraic features of the arguments.