

ON VARIETIES WITH ONE APPARENT DOUBLE POINT

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ABSTRACT. A smooth irreducible n -dimensional variety $X_n \subset \mathbb{P}^{2n+1}$ is said a *variety with one apparent double point*, briefly VOADP, if through a general point of \mathbb{P}^{2n+1} there passes a unique secant line to X_n or equivalently if a general projection into \mathbb{P}^{2n} has a unique double point as its singularity.

This condition imposes severe restrictions on the geometry of X_n (e.g. VOADP's are linearly normal and regular, $q(X) = h^1(\mathcal{O}_{X_n}) = 0$). The classification of OADP-curves is trivial, the unique being the twisted cubic, while the classification of OADP-surfaces was stated by Severi and completed by the author, see [Sv] and [Ru]. We will present the complete classification of OADP 3-folds, [CMR], and of n -dimensional OADP varieties of degree $d \leq 2n + 4$, [AR]. Moreover, we will illustrate how the geometrical properties of VOADP varieties cleared the path to the proof of interesting results in the classification theory of varieties (e. g. complete classification of varieties $X_n \subset \mathbb{P}^r$ of degree $d \leq r$, [AR] and Ionescu) and of general properties of special kinds of varieties (e.g. if $X_n \subset \mathbb{P}^{2n+1}$ is the scheme-theoretic intersection of quadrics having "enough linear syzygies", then X_n is rational and arithmetically Cohen-Macaulay, [AR]). We will (try to) discuss open problems and conjectures coming from classical papers on the subject, [Br].

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