Automorphism groups of maximal curves over finite fields

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Let X be a projective, geometrically irreducible, non-singular, algebraic curve defined over a finite field F of order q2 .If the number of F-rational points of X attains the Hasse–Weil upper bound, then X is called an F-maximal curve. Lachauds theorem stating that the quotient curve of X with respect to any subgroup of the F-automorphism group Aut(X) of a F-maximal curve is still an F-maximal curve. This offers a wide possibility to derive new F-maximal curves from a known one, and hence it gives a strong motivation for the study of F-automorphism groups of an F-maximal curve X. Let G be a subgroup of Aut(X) of a F-maximal curve. For a prime divisor p of the order of G, let H be the normal subgroup of G generated by all p-elements in G. For q even, the following classification theorem will be proved.

Theorem.

Let q be even, and let G have even order. Then one of the following cases occurs according as G fixes no or just one F-rational point of X. Either H isomorphic to one of the following groups PSU(3,22h), or SU(3,22h,), Sz(2h), h odd, SL(2,2h), or H is a Frobenius group such that the Frobenius kernel consists of all elements of odd order in H and the Frobenius complement is an abelian group with cyclic Sylow 2-group. 2)] H is a Sylow 2-group, and G is the semidirect product of H by the subgroup O(G) consisting of all elements of odd order in G. Each one of the above cases is known to occur for some values of h, apart from the possibility that H is isomorphic to SU(3,22h).